The Direct Method from Thermodynamics with Finite Speed used for Performance Computation of quasi-Carnot Irreversible Cycles.

I.Evaluation of coefficient of performance and power for refrigeration machines with mechanical compression of vapour

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This paper is dealing with the performances (Coefficient of Performance, COP and Power) evaluation for a quasi-Carnot irreversible cycle, namely the Irreversible Refrigeration Cycle with Mechanical Compression of Vapour. The present computation scheme is based on recent developments of the Direct Method from Finite Speed Thermodynamics (FST). The Direct Method consists in analyzing any irreversible cycle, step by step, by writing the corresponding equation of the First Law of Thermodynamics for finite speed processes and integrating it on the whole cycle, for each process. The First Law expression for finite speed processes includes three of the main sources of internal irreversibilities, namely: finite speed interaction between the piston and the gas/vapour, friction due to the finite piston speed within the cylinder, throttling processes in the valves. The new expression of the First Law for processes with finite speed is used here in order to get equations that relate the vapour properties for each irreversible process that occurs with finite speed of the piston in the compressor, in a Refrigeration Machine Cycle with Mechanical Compression of Vapour. These equations are used later for calculating entirely analytically the performances of the cycle, as a function of the finite speed of the piston and also as a function of other parameters such as: vapour temperatures and pressures in the evaporator, respectively in the condenser, compression ratio and mass flow rate. Note that the present analysis will differ from the previously reported one [1], because here the real properties of vapour in the cycle will be considered, for the first time in using the Direct Method, by using a correction for departure of vapour behaviour from the behaviour of perfect gases. The previous analysis used only the perfect gas properties. Therefore, these results can be used for optimization and design of refrigeration machines and heat pumps.

Keywords: reversed irreversible quasi-Carnot cycle, finite speed processes, Direct Method, refrigeration machines with finite speed, Thermodynamics with Finite Speed

The optimization of the Carnot cycle is a topic previously developed by the authors for Carnot and Stirling cycles [2-4]. The papers [2, 3] present analysis models based on the Direct Method and the First Law of Thermodynamics for processes with finite speed [5, 6]. In these papers, the study was further developed for an irreversible Carnot cycle with perfect gas as working fluid, achieved in four separate machines (an isothermal expansion machine at T_H, an adiabatic expansion machine, an isothermal compression machine at T_L and an adiabatic compression machine) that are connected through tubes and valves, keeping the expansion ratio constant during the isothermal process at the high temperature. The analytical results were applied for a particular set of engine parameters for which optimum piston speed corresponding to maximum power and optimum speed corresponding to maximum efficiency have been found. This sort of computation could help the designer to improve the performances of such machines. Recently, a similar model was developed for a Carnot cycle refrigeration machine. Also, an attempt of validation was made by using experimental data available for a real operating refrigeration machine [4].

The present paper analyses a reversed irreversible quasi-Carnot cycle with vapour (the cycle of Refrigeration Machine with Mechanical Compression of Vapour), starting from the previous works [4] where a reversed irreversible finite speed Carnot cycle with perfect gas was studied.

The objective of this approach, in comparison with the others [4], is to take into account the essential differences between the behaviour of vapour in comparison with perfect gases, and to analyze the necessary changes in the model in order to develop a new methodology for calculating fully analytical the irreversibilities (entropy generation) and performances (COP and power) of such a cycle.

The development of this new methodology can lead to completely analytically sensitivity and optimization studies of these machines, without using data from tables or software like Engineering Equation Solver (EES).

Applying the Direct Method to an irreversible cycle consists in using the mathematical expression of the First Law of Thermodynamics for processes with finite speed for each process and integrating it throughout the whole cycle. This leads to equations of irreversible processes of the cycle including the finite speed and other characteristic parameters for the cycle (pressure ratio, temperature ratio, etc.). Finally, based on these equations one gets the analytical expressions of the machine performances (COP and power). The mathematical expression of the First Law

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of Thermodynamics for processes with finite speed, previously developed [5, 6] includes the main sources of internal irreversibilities. It is used here for equation derivation of each irreversible process taking place with finite speed in a reversed irreversible quasi-Carnot cycle machine.

This paper and the following one (part II) define a quasi-Carnot cycle as any reversible or irreversible cycle, direct or reversed, which departs a "little bit" from a Carnot Cycle. Thus, Rankine and refrigeration cycles with mechanical compression of vapour (and corresponding heat pump cycles) are quasi-Carnot cycles, because they differ from a Carnot cycle (2-adiabatic and 2-isothermal processes) just on the small portion which is only isobaric (at high pressure) and not isothermal. But, let's not forget, that an important part of this isobaric process is also isothermal (in the domain of wet saturated vapour) and all the other 3 processes are exactly like in the Carnot cycle, respectively one entirely isothermal (at low pressure) and two adiabatic processes.

In this paper the pressure ratio p_2/p_1 (exit-entry of the compressor) replaces the volumetric compression ratio, previously used [4]. Also the deviation of vapour behaviour from the perfect gas is considered (fig. 1 and fig. 2).

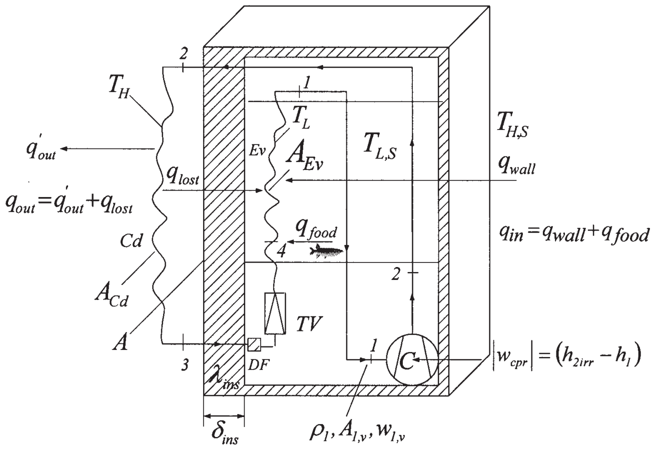

Fig.1. Vapour Compression Refrigeration Machine: C - Compressor, Cd - Condenser, DF - Dehydrator
TV - Throttling Valve, Ev – Evaporator

For the first time, the Direct Method is applied here to a reversed quasi-Carnot irreversible cycle with vapor, as a vapour compression refrigeration machine. A similar study was previously done for an irreversible Rankine cycle with finite speed [1], illustrating analytically and graphically the deviations of the optimized performances against the results achieved by Curzon – Ahlborn [7] in the Thermodynamics with Finite Time. Also the paper [1] have contained an example of a comparative study of the two methods, in order to illustrate how the expressions of *efficiency* and *power* of the cycle are different and how they can be used to evaluate the cycle performance closer to the real operation one. That comparison have emphasized that only the Direct Method from Thermodynamics with Finite Speed (TFS) can do that, by taking into account the internal irreversibilities from the beginning (in the equation of *The First Law for processes with finite speed* and subsequently in all the equations deriving from it). The Thermodynamics with Finite Time (TFT) does not consider the internal irreversibilities, from the beginning of its development [7], in the so called "endoreversible cycles". This was the most important reason for criticizing it (TFT) and its results by Moran [8, 9] and Giftopulous [10]. Thus, by "neglecting" the internal irreversibilities (which are in fact very important quantitatively) the results of TFT can not be used by engineers in real design and optimization of thermal machines. That was the very clear message of Moran and Giftopulous critics [8-10].

Fundamental Equations from Thermodynamics with Finite Speed [2-6, 12-16]

The Performances (*COP* and *power*) for a quasi-Carnot refrigeration cycle with irreversible processes are computed based on the *First Law of Thermodynamics for processes with finite speed* [5, 6, 11, 12]:

$$dU = \delta Q_{irr} - p_i\left(1 \pm \frac{aw_p}{c} \pm \frac{f\,\Delta p_f}{p_i} \pm \frac{\Delta p_{th}}{p_i}\right) dV \qquad (1)$$

where the work exchange in irreversible processes is:

$$\delta W_{irr} = p_i\left(1 \pm \frac{aw_p}{c} \pm \frac{\Delta p_f}{p_i} \pm \frac{\Delta p_{th}}{p_i}\right) dV \qquad (2)$$

Note that in Thermodynamics with Finite Speed (TFS) fundamental equations, (1) and (2), there are three important differences in comparison with Reversible Thermodynamics: a) - the new property p_i –instantaneously average pressure, b) - the parenthesis in the second term which takes into account the causes of irreversibilities, c) - the fact that in eq. (1) there is a factor f which does not appear in the eq. (2). In the Reversible Thermodynamics the second term in the First Law Expression is the reversible work (p . dV), which is the same if referring to the system or to the surroundings. In Irreversible Thermodynamics with Finite Speed (TFS) the work introduced in the system is different in comparison with "external work" done by the surroundings (or received by surroundings), because of the friction. Thus, in eq. (2) that expresses the irreversible external work done on the System (or by the System) by surrounding, does not appear the factor *f* which appears only in eq. (1). This "external work" from eq. (2) is the essential one to compute in comparison with the "internal work" from eq. (1), because it gives the chance to get an analytical expression of the power (developed or consumed) of any real operating thermal machine.

Each term in parenthesis of eq. (1) and (2) takes into account one type of internal irreversibility [5, 6, 11], as follows:

aw_p / c - contribution of finite speed of the piston, where $c=\sqrt{3RT}$; $a=\sqrt{3k}$; $k=c_p/c_v$ - adiabatic exponent;

$\Delta p_f / p_i$ - contribution of friction between mechanical parts;

$\Delta p_{th} / p_i$ - contribution of throttling process through the valves;

where p_i - is a new concept in comparison with Reversible Thermodynamics, namely the *instantaneously average pressure* in the system.

The factor f shows the part of the friction heat that remains inside the system, $0 \le f \le 1$. The case $f = 0$ corresponds to the case when all the friction heat is „lost" towards the surroundings (at the cold source); the case $f = 1$ is the other extreme, when all friction heat remains inside the system.

In equations (1) and (2), the sign (+) is used for compression and the sign (-) is used for expansion.

The mechanical friction and throttling losses are expressed in a similar manner to the case of internal combustion engines from Heywood [11], adapted by us in an appropriate way to be included in the expression of the *First Law of Thermodynamics with finite speed* applied for any piston-cylinder machines [5, 6, 12]. Thus, the application of the Direct Method in a similar way to Stirling machines, piston compressors, piston detentors and internal combustion engines provides for the first time a "unified treatment" of all of these types of irreversible machines. The expressions of these losses are:

$$\Delta p_f = A' + B' w_p \ ; \quad \Delta p_{th} = C w_p^2 \qquad (3)\text{-}(4)$$

where:

A'=0.94, ; B'=0.045 and C=0.0045 [9].

Note that the expression of the First Law of Thermodynamics for processes with finite speed, eq. (1), has been derived by the authors [5, 6, 12-16] by taking into account the irreversibilities introduced by the Second Law of Thermodynamics, caused by the piston finite speed. Therefore, eq. (1) is the essence of Thermodynamics with Finite Speed, by combining the First Law with the Second Law for irreversible processes generated by finite speed. Any research studying irreversible cycle should start with this equation, in order to take into account the internal irreversibilities. Hence, a strong tendency of "unification" of the two Irreversible Thermodynamics branches, respectively with *finite speed* (TFS) and with *finite time* (TFT) by using eq. (1) and (2) from TFS is in progress [17, 18].

Application of the Direct Method to the irreversible quasi-Carnot cycle refrigeration machine

The aim of this paper is to compare the reversible cycle $1\text{-}2_r\text{-}3\text{-}4_r\text{-}1$ (fig. 2) with the irreversible one with finite speed $1\text{-}2_{irr}\text{-}3\text{-}4_{irr}\text{-}1$, from the point of view of *COP* and *power*. Equations (1) and (2) can be integrated (analytically) for any processes in an irreversible cycle with finite speed in order to obtain the process equations and also the expressions for the irreversible work and heat exchange in those processes. For the irreversible cycle with finite speed from figure 2, eq. (1) is integrated only for the irreversible adiabatic process $1\text{-}2_{irr}$, in the compressor. Thus, the equation of irreversible adiabatic compression in the compressor is obtained. This equation will contain the origin of the internal irreversibilities, namely: the finite speed of the piston and the friction between piston and cylinder. Based on this equation, the temperature in the state 2_{irr}, namely T_{2irr} can be computed. Furthermore, the superheated vapor properties, h_{2irr} and s_{2irr}, as functions of T_{2irr} and p_2 become available by using the table with liquid and superheated vapour properties. These properties are necessary for computation of the work consumed in the compressor.

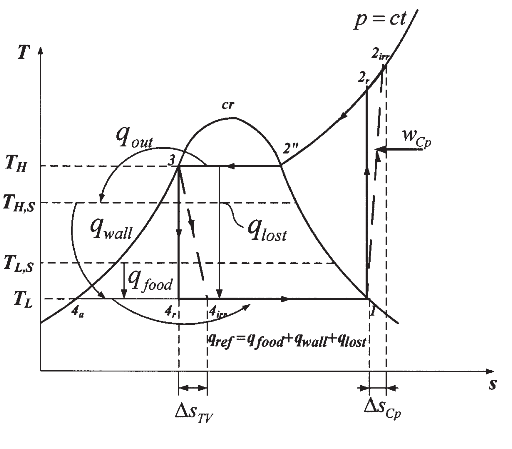

Fig. 2. Comparison of two Vapour Compression Refrigeration Machine Cycles in T-s diagram, seen as one reversed reversible quasi- Carnot cycle $1\text{-}2_r\text{-}3\text{-}4_r$, and the other one a reversed irreversible quasi-Carnot cycle $1\text{-}2_{irr}\text{-}3\text{-}4_{irr}$.

Nevertheless the use of tables is not our aim, as we want to obtain a complete analytical scheme of computation of the irreversible cycle $1\text{-}2_{irr}\text{-}3\text{-}4_{irr}$, in a similar way to the one of the Reversible Classical Thermodynamics, and the previously one done for Stirling machines [2]. Each process in the irreversible quasi-Carnot cycle shown in figure 2 occurs in a separate component: compressor ($1\text{-}2_{irr}$), condenser ($2_{irr}\text{-}3$), throttling valve ($3\text{-}4_{irr}$), evaporator ($4_{irr}\text{-}1$) (fig. 1). The resulting four components are assumed to be connected by tubes. In order to derive an expression easy to apply for expressing the internal entropy generation, COP_{irr} and power, only a new equation of the irreversible adiabatic processes with finite speed in the compressor ($1\text{-}2_{irr}$) is needed here (fig. 2). In the total irreversible process $3\text{-}4_{irr}$, the enthalpy is constant: $h_3 = h_{4irr,}$ and thus, another "new" equation of this process is not needed. But aiming to get a total analytical computation scheme the properties h and s on the limiting curves are expressed as function of pressure and temperature.

Let's take the irreversible adiabatic processes $1\text{–}2_{irr}$ (fig. 2). This is an irreversible process with 3 types of irreversibility causes: finite speed, friction and throttling. The First Law for processes with finite speed, eq. (1), is used and integrated by applying the Direct Method in order to find these equations. Assuming for the first approximation the hypothesis that the working fluid is a perfect gas and the adiabatic process condition is imposed, $\delta Q_{irr} = 0$, it results from eq. (1):

$$mc_v dT = -p\left(1 \pm \frac{a w_p}{\sqrt{3RT}} \pm \frac{f\,\Delta p_f}{p} \pm \frac{\Delta p_{th}}{p}\right) dV \qquad (5)$$

Equation (5) could be integrated in different assumptions in order to avoid cumbersome calculations. The simplest method of integration is described below. We denote the parenthesis that contains the irreversibility causes with $B = const = f\left(T_{med,1-2}, p_{med,1-2}, w_p\right)$, computed with average temperature $T_{med,1\text{-}2}$ and average pressure $p_{med,1\text{-}2}$ during the duration time of the process $1\text{-}2_{irr}$:

$$B = 1 \pm \frac{a w_p}{\sqrt{3RT_{med,1-2}}} \pm \frac{f \cdot \Delta p_f}{p_{med,1-2}} \pm \frac{\Delta p_{th}}{p_{med,1-2}}, \qquad (6)$$

where the average temperature is expressed as:

$$T_{med,1-2} = \frac{T_1 + T_2}{2} \qquad (7)$$

In order to estimate T_2 needed in eq. (7) we assume, in a first approximation, that $T_2 \cong T_{2r}$.

The equation for the vapour reversible adiabatic process $1\text{-}2_r$, yields:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k'-1}{k'}} = \lambda_p^{\frac{k'-1}{k'}} \qquad (8)$$

where: $\lambda_p = p_2 / p_1$ and k' is a corrected adiabatic exponent, which takes into account the difference between perfect gas and vapour of R134a, (fig. 3). This corrected k' exponent was obtained comparing T_2 computed with eq. (8), and T_{2r} computed based on constant entropy in the reversible adiabatic process $1\text{-}2_r$, and using tables data for vapour in state 2_r. As result of this computation figure 3 was plotted and the corresponding analytical formula for k' was derived.

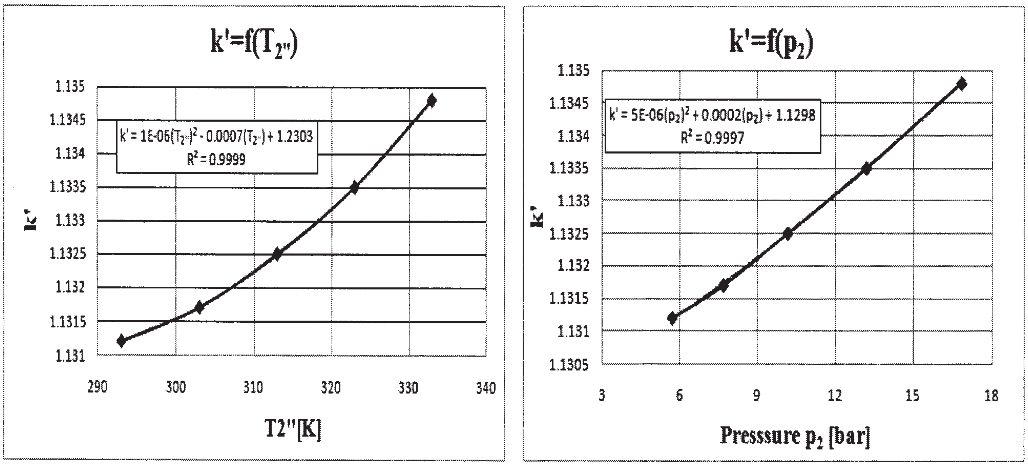

Fig. 3. Corrected adiabatic exponent k' variation versus the saturation vapour temperature $T_{2"}$, respectively pressure p_2.

For evaluation of $p_{med,\,1\text{-}2}$ we use the arithmetic average between initial and final pressures.

$$p_{med,1-2} = \frac{p_1 + p_2}{2} = \frac{p_1}{2}\left(1 + \frac{p_2}{p_1}\right) = \frac{p_1}{2}\left(1 + \lambda_p\right) \quad (9)$$

Upon substitution eqs. (8) and (9) in eq. (6), it results:

$$B = 1 \pm \frac{a w_p}{\sqrt{1.5 R T_1 \left(1 + \lambda_p^{\frac{k'-1}{k'}}\right)}} \pm \frac{2f\left(A' + B' w_p\right)}{p_1 (1 + \lambda_p)} \quad (10)$$

Note that eq. (10) takes into account only the contribution of the finite speed and friction. The throttling into the valves of the compressor will be taken into account separately.

Once the coefficient *B* is expressed as a function of the piston speed and the other gas parameters, we proceed with a variable separation in eq. (5):

$$\frac{m c_v' dT}{pB} = -\, dV \quad (11)$$

where the pressure is expressed from the state equation:

$$p = \frac{mRT}{V}. \quad (12)$$

By taking into account that a corrected specific heat c_v' which depends on k' (the *corrected adiabatic exponent)* is used in eq. (11):

$$c_v' = \frac{R}{k'-1}, \quad (13)$$

equation (11) becomes:

$$\frac{1}{B(k'-1)} \frac{dT}{T} = -\frac{dV}{V}. \quad (14)$$

This equation is different in comparison with the differential equation of adiabatic processes from Clasical Thermodynamics, because of two terms, *B* and *k'*. The term *B* takes into account the internal irreversibilities as function of the speed, w_p, and the term *k'* takes into account the departure of the superheated vapour in the compressor exit from the perfect gas behavior. All equations deriving from eq. (14) will contain these two „corrections", and they are important results of the Direct Method where this equation is integrated and the results are used for performances computation of the irreversible cycle that is studied.

By integrating eq. (14) for the irreversible adiabatic process $1 - 2_{irr}$, one gets:

$$ln \frac{T_{2_{irr}}}{T_1} = -B(k'-1) ln \frac{V_{2_{irr}}}{V_1} = ln \left(\frac{V_1}{V_{2_{irr}}}\right)^{B(k'-1)}, \quad (15)$$

which leads to the following equations for the irreversible adiabatic process with finite speed and friction:

- in coordinates T-V:

$$T_1 V_1^{B(k'-1)} = T_{2irr} V_{2irr}^{B(k'-1)} \quad (16)$$

- in coordinates p-V:

$$p_1 V_1^{B(k'-1)+1} = p_{2irr} V_{2irr}^{B(k'-1)+1} \quad (17)$$

- in coordinates T-p:

$$\frac{T_1}{T_{2irr}} = \left(\frac{p_1}{p_{2irr}}\right)^{\frac{B(k'-1)}{1+B(k'-1)}} \quad (18)$$

Equation (18) provides the temperature T_{2irr}. After getting the correlations between the specific enthalpy, h, and the specific entropy, s, as function of T on the isobaric process $2"\text{-}2_{irr}$, one can get immediately h_{2irr} and s_{2irr}, necessary for computation of the irreversible work needed by the compressor $(h_{2irr} - h_1)$. Since $1\text{-}2_{irr}$ is a compression process, the (+) sign appears in the analytical expression of B.

In this way the analytical expressions for the coefficient of performance, COP, and power are obtained, analyzing the successive influence of all the five internal losses.

The COP which takes into account only the finite speed of the piston in the compressor, COP_1 is:

$$COP_1 = \frac{h_1 - h_{4r}}{\left(h_{2irr}\right)_w - h_1} \quad (19)$$

and the corresponding power is:

$$P_{irr\,1} = \dot{m}\left(\left(h_{2irr}\right)_w - h_1\right) \quad (20)$$

where the mass flow rate is:

$$\dot{m} = \rho_1 \frac{\pi D^2}{4} w_p \quad (21)$$

with: ρ_1, the *density of vapour* in state 1 (fig. 2), $\rho_1 = \frac{1}{v_1}$; v_1, the specific volume in state 1 (fig. 2), and, D, the diameter of the piston (fig. 4)

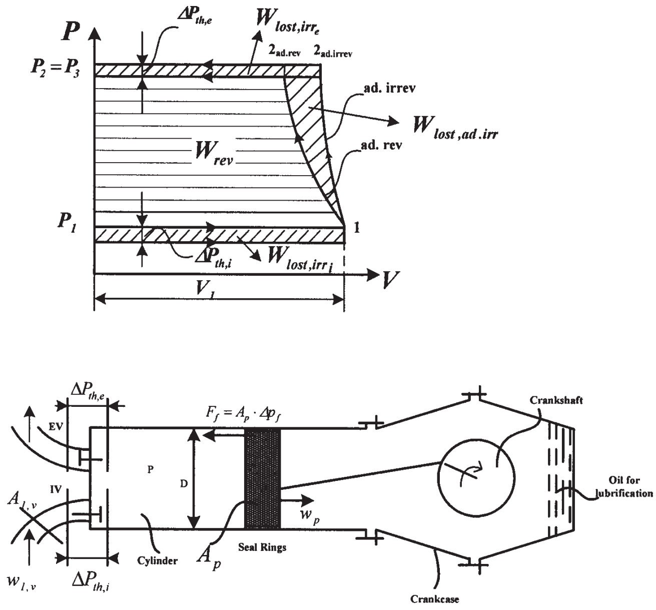

Fig. 4. Reversible and irreversible pV diagram of the ideal compressor

The COP which takes into account the finite speed of the piston and the friction in the compressor, COP_{II}, is:

$$COP_{II} = \frac{h_1 - h_{4r}}{(h_{2irr})_{w,f} - h_1} \tag{22}$$

and the corresponding power is:

$$P_{irr\,II} = \dot{m}\left((h_{2irr})_{w,f} - h_1\right) \tag{23}$$

The COP which takes into account the finite speed of the piston, the friction in the compressor and the throttling in the throttling valve COP_{III} is:

$$COP_{III} = \frac{h_1 - h_{4irr}}{(h_{2irr})_{w,f} - h_1} \tag{24}$$

and the corresponding power remains $P_{irr\,II}$, as the compressor and the throttling valve are independent machine components.

The COP which takes into account the finite speed of the piston, the friction in the compressor, the throttling in the throttling valve, TV (fig.1) and the throttling in the compressor valves, EV and IV (fig.4), COP_{IV} is:

$$COP_{IV} = \frac{h_1 - h_{4irr}}{(w_{Cp})_{w,f,th_{Cp}}} = \frac{h_1 - h_{4irr}}{(w_{Cp})_{w,f} + w_{th_{Cp}}} \tag{25}$$

where:

$$(w_{Cp})_{w,f,th_{Cp}} = \underbrace{\left[(h_{2irr})_{w,f} - h_1\right]}_{(w_{Cp})_{w,f}} + \underbrace{\Delta p_{th,e} \cdot v_{2irr} + \Delta p_{th,i} \cdot v_1}_{w_{th_{Cp}}} \tag{26}$$

and the corresponding power is:

$$P_{irr\,III} = \dot{m} \cdot (w_{Cp})_{w,f,th_{Cp}} \tag{27}$$

The COP which takes into account the finite speed of the piston, the friction in the compressor, the throttling in the compressor valves (EV and IV), the throttling in the throttling valve (TV), but also the heat losses q_{lost} from the (fig. 1), between heat sources, COP_V, is computed with the following equations. Starting with the definition of COP one gets:

$$COP = \frac{q_u}{w_{Cp}} \quad \Rightarrow \quad COP_V = \frac{(h_1 - h_{4irr}) - q_{lost}}{\left((h_{2irr})_{w,f} - h_1\right) + w_{th_{Cp}}} \tag{28}$$

where:

$$q_u = q_{ref} - q_{lost} = (h_1 - h_{4irr}) - q_{lost} \tag{29}$$

$$q_{ref} = q_{lost} + q_{in} = (h_1 - h_{4irr}) \tag{30}$$

$$q_{lost} = \frac{\dot{Q}_{lost}}{\dot{m}} = \frac{KA(T_H - T_L)}{\dot{m}} \tag{31}$$

with: K - the overall heat transfer coefficient:

$$K = \left[\frac{1}{\alpha_e} + \frac{\delta_{ins}}{\lambda_{ins}} + \frac{1}{\alpha_i}\right]^{-1} \tag{32}$$

and A is the average area between the corresponding ones of the evaporator and condenser:

$$A = \frac{A_{Ev} + A_{Cd}}{2} \tag{33}$$

Eq. (28) can be written in the form:

$$COP_V = \frac{h_1 - h_{4irr}}{\left((h_{2irr})_{w,f} - h_1\right) + w_{th_{Cp}}} - \frac{q_{lost}}{\left((h_{2irr})_{w,f} - h_1\right) + w_{th_{Cp}}} =$$

$$= \underbrace{\frac{h_1 - h_{4irr}}{\left((h_{2irr})_{w,f} - h_1\right) + w_{th_{Cp}}}}_{COP_{w,f,th_{Cp},th_{TV}}} \underbrace{\left[1 - \frac{q_{lost}}{h_1 - h_{4irr}}\right]}_{\eta_{IIQlost}} \tag{34}$$

Based on this identification of the terms, one gets:

$$COP_V = COP_{IV}\ \eta_{II,Q_{lost}} \tag{35}$$

where:

$$\eta_{II,Q_{lost}} = 1 - \frac{\left(\frac{A_{Ev} + A_{Cd}}{2}\right)(T_H - T_L)}{\dot{m}\left(\frac{1}{\alpha_e} + \frac{\delta_{ins}}{\lambda_{ins}} + \frac{1}{\alpha_i}\right)(h_1 - h_{4irr})} \tag{36}$$

$\eta_{II,Qlostl}$ - the Second Law Efficiency generated by heat losses between the two heat sources.

Finally the following equation yields:

$$COP_V = \frac{(h_1 - h_{4irr}) - \dfrac{\left(\frac{A_{Ev} + A_{Cd}}{2}\right)(T_H - T_L)}{\dot{m}\left(\frac{1}{\alpha_e} + \frac{\delta_{ins}}{\lambda_{ins}} + \frac{1}{\alpha_i}\right)}}{\left((h_{2irr})_{w,f} - h_1\right) + w_{th_{Cp}}} \tag{37}$$

By using the expressions from the Table 1, with properties of the vapour in the main states of the cycle, and introducing in eq. (37), the final analytical formula for *COP* is:

$$COP_V = \frac{15846\,ln(p_1) + 3 \cdot 10^{-8}(p_3)^2 + 0.1086(p_3) + 26444 - A_1}{B_1 - 15846\,ln(p_1) - 120061 + w_{th_{Cp}}} \tag{38}$$

where
$$A_1 = \frac{11.72 \cdot 10^8 A_{Ev} \dfrac{(p_2)^{\frac{B(k'-1)}{B(k'-1)+1}}}{(p_1)^{\frac{10.7582B(k'-1)-1}{B(k'-1)+1}}}}{\left(\frac{1}{\alpha_e} + \frac{\delta_{ins}}{\lambda_{ins}} + \frac{1}{\alpha_i}\right)\frac{\pi D^2}{4} w_p} \tag{39}$$

Table 1
REFRIGERANT PROPERTY EXPRESSIONS IN THE TWO CYCLES MAIN STATES

STATE 1	STATE 2r	STATE 2irr	STATE 3	STATE 4a	STATE 4r	STATE 4irr
$T_1 = 74.582(p_1)^{0.1035}$	$T_{2r} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{k'-1}{k'}}$	$T_{2irr} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{B(k'-1)}{B(k'-1)+1}}$	$T_3 = -2\cdot 10^{-11}(p_3)^2 + 8\cdot 10^{-5}(p_3) + 253.87$	$T_{4a} = T_1$	$T_{4r} = T_1 = 74.582(p_1)^{0.1035}$	$T_{4irr} = T_1 = 74.582(p_1)^{0.1035}$
p_1 – given	p_{2r} – given	$p_{2irr} = p_{2r}$	$p_3 = p_{2irr} = p_{2r}$	$p_{4a} = p_1$	$p_{4r} = p_1$	$p_{4irr} = p_1$
$s_1 = -15.51\ln(p_1) + 1128.8 = 0.0045(T_1)^2 - 3.0762(T_1) + 1436.2$	$s_{2r} = -0.0032(T_{2r})^2 + 5.3361(T_{2r}) - 428.19$	$s_{2irr} = -0.0032(T_{2irr})^2 + 5.3361(T_{2irr}) - 428.19$	$s_3 = -10^{-10}(p_3)^2 + 0.0004(p_3) + 113.3 = 4.725(T_3) - 1084.8$	$s_{4a} = -10^{-10}(p_1)^2 + 0.0004(p_1) + 113.3 = 4.725(T_1) - 1084.8$	$s_{4r} = s_3 = -10^{-10}(p_3)^2 + 0.0004(p_3) + 113.3 = 4.725(T_3) - 1084.8$	$s_{4irr} = s_{4a} + (s_1 - s_4)\left(\frac{h_3 - h_{4a}}{h_1 - h_{4a}}\right)$
$h_1 = -1.3367(T_1)^2 + 1305.8(T_1) - 6407.8 = 15846\ln(p_1) + 51610$	$h_{2r} = 1085(T_{2r}) - 68451$	$h_{2irr} = 1085(T_{2irr}) - 68451$	$h_3 = -3\cdot 10^{-8}(p_3)^2 + 0.1086(p_3) + 25166 = 2.1886(T_3)^2 + 124.47(T_3) - 145250$	$h_{4a} = -3\cdot 10^{-8}(p_1)^2 + 0.1086(p_1) + 25166 = 1450.6(T_1) - 1084.8$	$h_{4r} = h_{4a} + x_4\ (h_1 - h_a)$	$h_{4irr} = h_3 = -3\cdot 10^{-8}(p_3)^2 + 0.1086(p_3) + 25166 = 1450.6(T_3) - 1084.8$
$v_1 = 8\cdot 10^{21}(T_1)^{-9.46} = 15715(p_1)^{0.981}$	$v_{2r} = v_1\left(\frac{T_1}{T_{2r}}\right)^{\frac{1}{k'-1}}$	$v_{2irr} = v_1\ \theta^{B(k'-1)}$	$v_3 = -2\cdot 10^{-17}(p_3)^2 + 2\cdot 10^{-10}(p_3) + 0.0007 = 2\cdot 10^{-8}(T_3)^2 - 9\cdot 10^{-6}(T_3) + 0.0018$	$v_{4a} = -2\cdot 10^{-17}(p_1)^2 + 2e^{-10}p_1 + 0.0007$	$v_{4r} = v_3\left(\frac{p_3}{p_4}\right)^{k'}$	$v_{4irr} = v_3\left(\frac{T_3}{T_1}\right)^{\frac{1}{B(k'-1)}}$
$x_1 = 1$			$x_3 = 0$	$x_{4a} = 0$	$x_{4r} = \frac{s_{4r} - s_{4a}}{s_1 - s_{4a}}$	$x_{4irr} = \frac{h_3 - h_{4a}}{h_1 - h_{4a}}$
$B = 1 \pm \frac{a w_p}{\sqrt{1.5RT_1\left(1+\lambda_p^{\frac{k'-1}{k'}}\right)}} \pm \frac{2f\,(A' + B' w_p)}{p_1\,(1+\lambda_p)}$				$k' = 10^{-6}(T_{2r})^2 - 0.0007(T_{2r}) + 1.2303 = 5\cdot 10^{-6}(p_{2r})^2 + 0.0002(p_{2r}) + 1.1298$		

and

$$B_1 = 80921.47\frac{(p_2)^{\frac{B(k'-1)}{B(k'-1)+1}}}{(p_1)^{\frac{8.6618B(k'-1)+1}{B(k'-1)+1}}}. \qquad (40)$$

The corresponding power remains P_{irrIII}, as the compressor work is not affected by the heat losses between sources.

In order to calculate the isentropic efficiency of the compressor, η_{is}, by using the definition and considering all internal irreversibilities above mentioned, the following expression yields:

$$\eta_{is} = \frac{h_{2r} - h_1}{(h_{2irr} - h_1) + w_{th_{Cp}}} =$$

$$\frac{80921.47\dfrac{p_2^{\left(\frac{k'-1}{k'}\right)}}{p_1^{\left(\frac{k'-1}{k'}-0.135\right)}} - 15846\ln(p_1) - 120061}{80921.47\dfrac{p_2^{\left(\frac{B(k'-1)}{k'}\right)}}{p_1^{\left(\frac{B(k'-1)}{k'}-0.135\right)}} - 15846\ln(p_1) - 120061 + w_{th_{Cp}}} \qquad (41)$$

Results

The calculations are done considering the following data for dimensions and properties: L=1m, D=0.05m, N_{pipes}=8, α_e = 7W / m²K, α_i = 5W / m²K, λ_{ins} = 0.044 W / mK, δ_{ins} = 0.1 m and $A_{Ev} = A_{Cd}$ = 0.176 m².

It was found that the irreversible adiabatic compression process can be described quantitatively by an adiabatic equation similar with the reversible one for perfect gases but corrected with an exponent which takes into account the difference between gas and vapour, denoted by k'. This new adiabatic exponent k' depends on $T_{2''}$ (or p_2) and is different of the reversible adiabatic exponent $k = 1.3$. In figure 3 this variation stands out.

By using the above derived (completely analytical) formulae for *COP* and *power*, the effect of internal irreversibilities progressively introduced on the cycle performances are illustrated in figure 5. Thus, the major reductions of COP are registered when the friction losses are considered (COP_{II}), respectively the throttling in the compressor valves (COP_{IV}). As expected, the power needed by the compressor increases with each new irreversibility. By comparing the variation of the two performances with the piston speed, clearly appears that small values of the piston speed provide economical operational regime, mainly from the power consumption reason. For example, when passing from 0.5m/s to 1m/s, the power increases twice, namely from 300W to 600W. As previously mentioned, the Power of the compressor does not depend neither on the expansion process (3-4ir), nor on heat losses between heat exchangers, but COP depends on the irreversibility introduced by each of these processes. This fact explains the existence of 6 curves for COP and only 4 curves for Power in figure 5. Thus, Power P_{II} corresponds to both curves of COP_{II} and COP_{III}, while Power P_{III} corresponds to both curves of COP_{IV} and COP_V.

To highlight the influence of all internal irreversibilities on the operation of a real refrigerator, we analyzed the results obtained by varying the temperature $T_H = T_{2''}$. (fig. 6)

It can be noticed that for a constant temperature of the cooled space T_L, and as the superheated saturated vapour temperature decreases, the work needed for the reversed quasi – Carnot cycle and the refrigeration efficiency, COP of the cycle decrease, so seemingly the refrigeration cycle is more efficient with the decrease of the temperature difference between sources, as expected.

Following figure 6 it is found that COP decreases with the increase of $T_{2''}$ and with the increase of the piston speed w_p. It seems that decreasing $T_{2''}$ is a good idea for a better design, in order to enlarge the COP. Unfortunately this

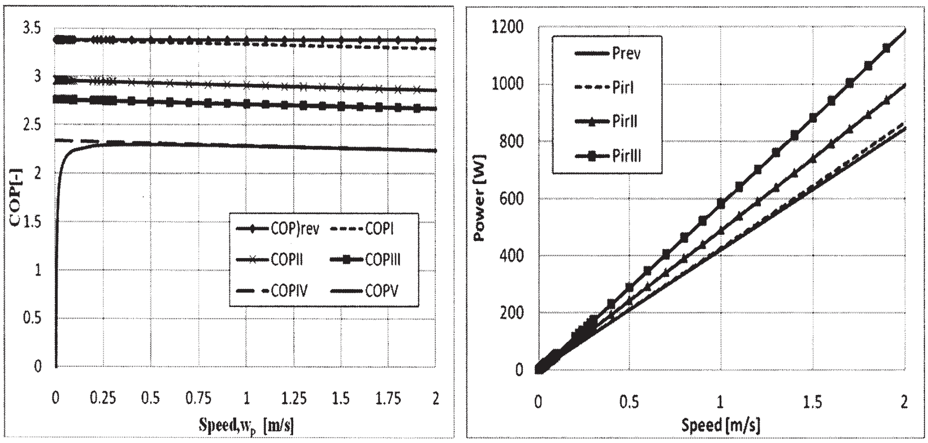

Fig. 5. The effect of different internal types of irreversibilities on COP and power (w_p=0÷ 2m/s)

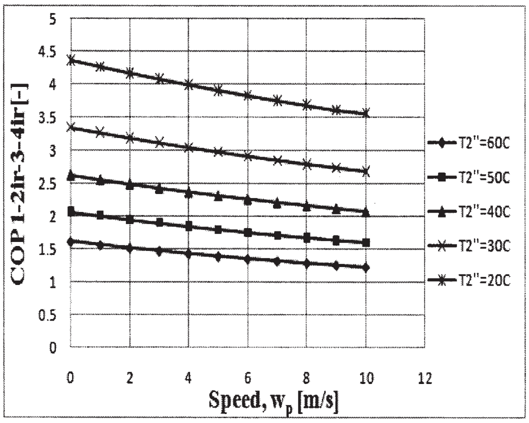

Fig. 6. The effect of piston speed w_p on COP (1-2_{irr}-3-4_{irr}) in the quasi-Carnot cycle for various saturation temperatures $T_{2"}$

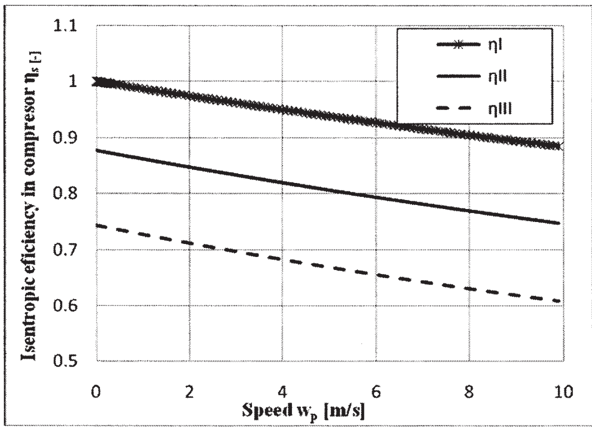

Fig. 8. Isentropic efficiency in the compressor as function of the piston speed, w_p

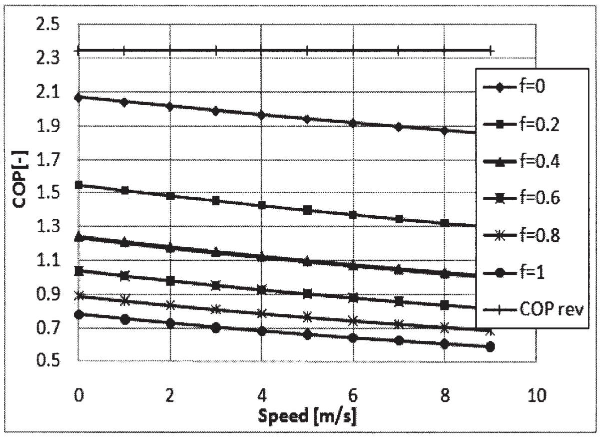

Fig. 7. The influence of the factor f on COP

conclusion is false because it was not yet taken into account the effect of external irreversibilities. For example, with the decrease of $T_{2"}$ the temperature differences increasingly lowers (at the hot source where heat is evacuated). This would entail to increase the surface of the heat exchanger of the condenser, which would involve additional costs for the construction of the machine and also larger storage space.

The influence of factor f, showing the part of the friction heat that remains inside the system, upon COP, is illustrated in figure 7. An important reduction of COP, about 3 times, is registered for a given value of the piston speed, when factor f value passes from 0 to 1.

Based on eq. (41) and by introducing successively different types of losses, a gradually reduction of the isentropic efficiency in the compressor is observed in figure 8. Note that the losses separating the three curves correspond to those mentioned for the power calculation, P_I to P_{III}.

Only through a combined analysis which takes into account both internal and external irreversibilities, combined with an economic and technical analysis, $T_{2"}$ can be optimized as well as the COP. This objective remains to be developed in a future paper. Also another goal could be a scheme for calculating large refrigerating machine that uses a piston detentor or a turbine instead of the throttling valve. It requires a similar analysis to that of the compressor, also based on the Direct Method, and is under development in our research group.

Conclusions

Performances calculation for a reversed irreversible cycle quasi-Carnot machine (Vapour Compression Refrigeration Machine) is presented. The paper develops an analysis of internal irreversibilities generation in a Mechanical Vapour Compression Refrigeration Machine, operating with finite speed. The results obtained by using the calculation scheme that was developed based on the Direct Method from Thermodynamics with Finite Speed, gave the chance to evaluate the irreversibilities and performances in a purely analytical manner. For the first time in the Direct Method analysis of a cycle, the essential difference between the vapour behaviour in comparison with perfect gases were taken into account. In developing the analytical model, irreversibilities produced during the adiabatic compression (1-2_{irr}), such as finite speed of the piston, friction, throttling in the compressor valves, and throttling in the adiabatic expansion (3-4_{irr}) in TV (fig.1) were considered. The analysis was done here completely analytic, which means that formulae for COP and Power, as function of the piston speed w_p in the compressor, and other parameters (compression ratio λ_p, vapour pressure and temperature, mass flow rate, etc) are derived. Also, it allowed a step by step sensitivity study with respect to the factors causing internal irreversibilities. Thus, the influence degree of each cause of irreversibility clearly appeared. Based on such calculation the designer has a chance to "see" where to intervene in order to improve the performances of the whole system. By taking into account other internal irreversibilities this scheme of computation can be further developed, and the final goal of validation achieved. Such a validated scheme of computation could help the optimal design of Refrigeration Machines and Heat Pumps.

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Nomenclature

A - area, m^2
a - coefficient (= √ 3k)
c - average molecular speed, $m\ s^{-1}$
c_p, c_v - specific heats, $J\ kg^{-1}\ K^{-1}$
COP - coefficient of performance
D - diameter, *m*
f - coefficient related to the friction contribution ∈ (0, 1)
h -specific enthalpy, $J\ kg^{-1}$
K - overall heat transfer coefficient, $W\ m^{-2}\ K^{-1}$
k - ratio of the specific heats
k' - corrected adiabatic exponent
m - mass, *kg*
m - mass flow rate, $kg\ s^{-1}$
N - number of pipes
p - pressure, *Pa*
Δp - pressure loss, *Pa*
Q - heat, *J*
q - heat lost per mass unit, $J\ kg^{-1}$
R - gas constant, $J\ kg^{-1}\ K^{-1}$
S - stroke, *m*
s - specific entropy, $J\ kg^{-1}K^{-1}$
T - temperature, *K*
U - internal energy, *J*
V - volume, m^3
v - specific volume, $m^3\ kg^{-1}$
ΔV - volume variation, m^3
w - specific work, $J\ kg^{-1}$
w_p - piston speed, $m\ s^{-1}$

Greek symbols

α - convection heat transfer coefficient, - $W\ m^{-2}K$
λ - thermal conductivity, $W\ m^{-1}\ K^{-1}$
λ_p - compression ratio in the compressor
δ - thickness of the wall insulation, *m*
ρ - density of vapor, $kg\ m^{-3}$
η_{II} - second law efficiency

Subscripts

Cp - compressor
Ev - evaporator
f - friction
H - hot-end of the machine
i - instantaneous
ins - insulation
irr - irreversible
L - cold-end of the machine
med - average
r -reversible
th -throttling
w -finite speed

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