# Design of Pressure Equipment Subject to Fatigue

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*The pressure equipment fatigue loading is analyzed. A general survey of the calculation of pressure vessels cyclically loaded in the Euronorma 13445 and in the ASME Code is presented. Based on the principle of critical energy it was developed a new method of calculation of pressure vessels subject to fatigue.*

*Keywords: pressure equipment, fatigue, deterioration, nonlinear behavior, principle of critical energy*

Fatigue is one of the most dangerous form of damage. .Fatigue damage only lead to a sudden catastrophic fracture of a pressure equipment or, generally, of a mechanical structure.

Cyclic loading with normal stresses (fig. 1) features maximum normal stress, $\sigma_{max,}$ minimum normal stress, $\sigma_{min}$, and cycle duration $t_{c,ó}$.

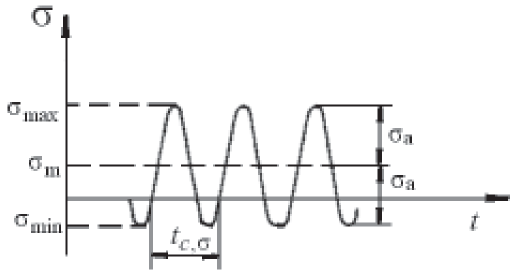


Fig. 1. Normal stress variation under cyclic loading

The features are defined as follows:

normal mean stress, $\sigma_m = 0.5(\sigma_{max} + \sigma_{min})$;
normal stress amplitude, $\sigma_a = 0.5(\sigma_{max} - \sigma_{min})$;
normal stress range, $\Delta\sigma = \sigma_{max} - \sigma_{min} = 2\sigma_a$ (1)
cycle asymmetry coefficient, $R(\sigma) = \sigma_{min}/\sigma_{max}$.

For cyclic loading with shear stresses it is used an analogous procedure by replacing σ with τ in relations 1. So, there are obtained:

mean shear stress, $\tau_m = 0.5(\tau_{max} + \tau_{min})$;
shear stress amplitude, $\tau_a = 0.5(\tau_{max} - \tau_{min})$;
shear stress range, $\Delta\tau = \tau_{max} - \tau_{min} = 2\tau_a$; (2)
cycle asymmetry coefficient, $R(\tau) = \tau_{min}/\tau_{max}$.

Practically, it is considered that loading has the following features [1]:

– it is static if the duration of stress increases from 0 to $\sigma_{max}$ is higher or at least equal to three times the period of individual structure vibrations, $t_p$;

– it is rapid, if duration has values ranging between $0.5t_p$ and $3t_p$;

– it is shockwise, if $t \le 0.5t_p$

The value of strength to fatigue fracture is determined by the asymmetry coefficient, *R*, the number of loading cycles, *n*, and the value of the stress rate, dσ/d*t* and dτ/d*t*.

The behaviour of materials cyclically loaded without taking into consideration the cracks is represented graphically as a Wöhler curve (fig. 2), which shows the dependence between maximum stress, $\sigma_{max}$, or stress amplitude, $\sigma_a$, and the number of loading cycles down to fracture, *N*. One writes *n* ($n \le N$) for the effective number of loading cycles and $N_0$ for the number of cycles that define the fatigue limit, $\sigma_{-1}$ or $\tau_1$.

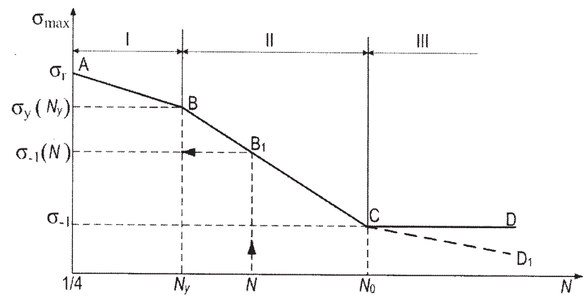


Fig. 2. Dependence of number of loading cycles, *N*, down to fracture, for a polished sample [2;3] of maximum stress (Wöhler's curve) under alternating symmetrical loading (semilogarithmic diagram)

On the fatigue Wöhler type curve (fig. 2) can be distinguished three domains:

I – domain of oligocyclic loading (of the reduced number of loading cycles), where $\sigma_y(N_y) \le \sigma_{max} \le \sigma_u(N)$ and $N \in [1/4; N_y]$;

II – domain of limited durability where $N \in (N_y; N_o)$ and $\sigma_{max} \in (\sigma_y(N_y); \sigma_{-1})$;

III – domain of unlimited loading, where $\sigma_{-1}$ is the fatigue limit and $\sigma_{max} \le \sigma_{-1}$.

Here $\sigma_y$ is the yield normal stress and $\sigma_u$ – the ultimate normal stress.

With a nummber of over $10^6$ cycles, for steels, the fatigue limit is constant. At present, in the welded joint of steel components one accept decrease in fatigue strength, ranging between $10^6$and $10^8$cycles. There are many materials whose fatigue strength decreses with a number of cycles exceeding $10^7$cycles [2]. At present, if a sample that did not fail after $10^7$cycles has infinite durability, which represents a convention imposed by economic prerequisites.

Generally speaking, the pressure equipment is loaded when pressure fluctuations occur while featuring a relatively small number of cycles, $n \approx 10^3 - 5\times10^4$ cycles. In these conditions, the critical number of cycles lies in the area of plastic state loading, the oligocyclic loading (fig. 2). This corresponds to $n < N_y$ and $\sigma_y < \sigma_{max} < \sigma_u$ .

The value of the total deterioration determined by a succession of cyclic loadings is calculated at present with

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Palmgren-Miner's law, according to which the total deterioration is [4;5]:

$$D_T = \sum_i D_i, \qquad (3)$$

where $D_i = \left(\frac{n}{N}\right)_i$ is the partial deterioration determined by loading with stress amplitude $\sigma_{a,i}$, a number of $n_i$ cycles.

When reaching the critical state $D_T = D_{cr}$, where $D_{cr} \leq 1$ is the critical deterioration. Value $D_{cr}$ depends on the loading conditions and the hypotheses made with respect to it. At present, the pressure equipment fatigue calculation is done differently in the different existing standards (ASME Code, Euronorma, British Standard etc.).

## Fatigue calculation of pressure equipment according to several standards

*Calculations according to Euronorme 13445 [6]*

According to this standard, along pressure variation, one can take into consideration the temperature variations and the variation of external loads if:

– the cyclic loads (other than pressure) occur simultaneously with the cyclic variation of pressure. In this case, the stress variation determined by cyclic loads are added to the stress determined by cyclic pressure variation; one calculates the total normal stress range,

$$\Delta \sigma_{tot} = \Delta \sigma (p) + \Delta \sigma(F), \qquad (4)$$

where $p$ is pressure, $F$ is taken as a generalized force;

– cyclic loads (other than pressure)act independently of cyclic pressure variation. In this case, the deterioration produced by variation $\Delta F$, cyclic loads, $D(\Delta F)$, which is added to the deterioration produced by cyclic pressure variation, $D(\Delta p)$, such as the total deterioration is:

$$D_t = D(\Delta F) + D(\Delta p). \qquad (5)$$

To survive it is necessary to fulfill the following condition,

$$Dt \leq D_{max}, \qquad (6)$$

where:

$$D_{max} = 0.8 \quad \text{for } 500 < n_{eq} < 1000;$$
$$D_{max} = 0.5 \quad \text{for } 1000 < n_{eq} < 10000; \qquad (7)$$
$$D_{max} = 0.3 \quad \text{for } < n_{eq} < 10000;$$

The number of equivalent cycles, $n_{eq}$, is the total number of pressure cycles that causes the same deterioration produced by the pressure variation $\Delta p$ in a number of $n$ cycles.

In case $\Delta p_i < p$, where p is the nominal pressure, the calculation of $n_{eq}$ uses the relation:

$$n_{eq} = \Sigma\, n_i \cdot (\Delta p_i / p_{max})^3, \qquad (8)$$

where $p_{max}$ is the maximum allowable pressure, calculated for the whole vessel under normal conditions. The standard is applicable to vessels with flawless welded joints or the limited flaws accepted in EN 13445-5:2009.

The influence of pressure fluctuations can be neglected if:

$$\Delta p < 5\% \, \Delta p_{max}, \qquad (9)$$

where $\Delta p = p_{max} - p_{min}$, is the pressure range.

The fatigue curve (fig. 3) expressed as dependence between stress variation $\Delta\sigma = \sigma_{max} - \sigma_{min} = 2\sigma_a$ and $N$, comprises two zones separated by $N_0$ ($N_0 = 2 \cdot 10^6$ for the base material; $N_0 = 5 \cdot 10^6$ for welds).

The fatigue curve for welds extends up to $N_0{}^4 = 1 \cdot 10^8$ cycles. One considers that the stress variations lower than

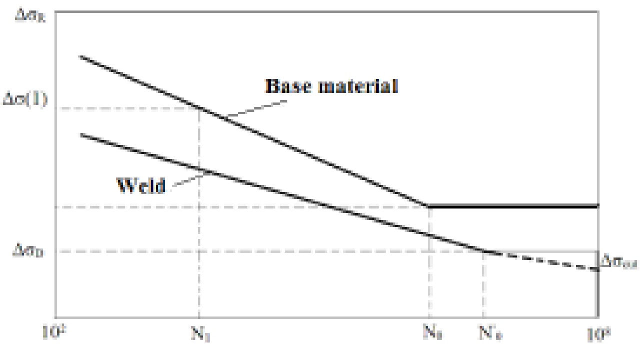


Fig. 3. Fatigue curves for base material and welding (qualitative) [6] .

$\Delta\sigma_{cut}$ do not cause fatigue deterioration and may be neglected. In this standard, the description of the fatigue curve is based on Basquin's law[7],

$$\sigma_a^m \cdot N = A, \qquad (10)$$

where $A$ and $m$ are the constants of the material undergoing fatigue. It is recommendable to use different relations for the welded joint and the base material [6].

For loading with several stress blocks one considers the successive application of normal stresses. The total fatigue deterioration is calculated with relation (3). The load is considered acceptable if $D_t \leq 1$. If this condition is not fulfilled, the design has to be rectified and a detailed analysis is carried out based on standard EN 13445, which consist of:

– calculation of cumulated deteriorations caused by different load amplitudes with relation (3), where $n$ is the number of loading cycles applied, while $N$ is the allowable number of loading cycles obtained from the fatigue design curve. The fatigue curve, as specified in the standard, includes the effect of mean stress;

– the calculation of the allowable number of loading cycles with relation: $N = N_{cr} / c_N$, where $N_{cr}$ is the number of loading cycles until the critical state has been reached, while $c_N = 10$ is a safety coefficient.

The value of $N$ is obtained directly from the fatigue curve (fig. 3). With this aim in view, one calculates the stress range $\Delta\sigma$ which is done differently for welding and the base material;

– calculation of the equivalent loading is done by using the Tresca theory of strength.

We deal with the base (unwelded) material analogously.

*Fatigue loading in the elasto-plastic domain*

Both for weld and the base material we consider that loading lies in the elasto-plastic domain if the variation of the structural pseudoelastic stress is two times greater than $R_{p,02/T}$, for ferritic steels and $R_{p,01/T}$, respectively, for austenitic steels, where T* is the average cycle temperature.

With loading in the elasto-plastic field the stress range is increased by multiplying it with a $k_e > 1$ factor –for stresses of mechanical nature and a $k_v$ factor – for stresses of thermal nature. The variation of the equivalent mechanical structural stress is:

$$\Delta\sigma_{struct,eq} = k_e \cdot \Delta\sigma_{eq,l}, \qquad (11)$$

where $\Delta\sigma_{eq,l}$ is the variation of the equivalent stress corresponding to the variation of the stress equivalent linear distribution.

It was accepted, from theoretical analysis or experimentall that the total strain variation (elastic plus plastic), there is no need to make any correction for plastic behaviour, a situation when the normal stress variation is calculated with Hooke's law,

$$\Delta\sigma = E \cdot \Delta\varepsilon_T, \tag{12}$$

where $E$ is the elasticity modulus and $\Delta\varepsilon_T$ is the total strain variation.

*Fatique calculation according to ASME Code, Section VIII, Division 2 [8]*

The calculation using this particular standard is based on curves $\sigma_n$-$\varepsilon_n$ (where $\sigma_n$ and $\varepsilon_n$ are the true normal stress and the true strain).

The analysis in the elastic domain ($\sigma<\sigma_y$) is based on the calculation for the components of the stress amplitude variation tensor, $\Delta\sigma_{ijk}$

– one calculates the allowable number of loading cycles, $N_k$, based on the value of the equivalent alternative stress amplitude for cycle $k$:

$$S_{alt,k} = 0.5 \cdot E_{yf} \cdot \Delta\varepsilon_{eff,k} \tag{13}$$

where $E_{yf}$ is the elasticity module from the fatigue curve. Relation (13) is correct only in the domain of linear-elastic behaviour;

– one calculates the deterioration produced in cycle k with relation:

$$D_{f,k} = \frac{n_k}{N_k} \tag{14}$$

where $N_k$ depends on $S_{alt,k}$;

– we repeat the same procedure for all the loading blocks and calculate the total deterioration,

$$D_T = \sum_{k=1}^{M} D_{f,k}. \tag{15}$$

The total deteriorations is compared with the maximum value, $D_{max} = 1$. The acceptance conditions is $D_T \leq 1$.

· Analysis in the *plastic domain* is carried out on the basis of the effective strain variation for cycle k, which also contains the sum of strain variation in the elastic state and strain variation in the plastic state;

– we calculate deteriorations for all the loading cycles and the total deterioration with relation (15).

The condition is $D_T \leq 1.0$. Analogously, we do the same for the welded joint.

The structure of relation (14) corresponds to linear-elastic behaviour. Actually, $\Delta\varepsilon_{eff,k} = \Delta\varepsilon_{e,k} + \Delta\varepsilon_{p,k}$, represents the elasto-plastic strain variation; this corresponds to the nonlinear behaviour of the material.

*Comments on present methods for fatique design*

According to standard EN 13445/2009, the fatigue diagrams include the possible effects of mean and residual stresses which might lead to overdimensioning. For example, in the case of symmetrical alternating loads, there are no mean stresses ($\sigma_m = 0$) as well as in the case when by de-stressing, the residual stresses have been annihilated or greatly reduced ($\sigma_{res} \approx 0$).

For loadings with several stress blocks, the standards do not take into consideration the actual existing residual stresses and the influence of the effective mean stress, as well as the order of loadings.

When doing the fatigue calculation according to ASMECode/2009, it is used the Ramberg-Osgood law for loadings in the plastic domain, which might lead to errors since strain in the plastic state (where behaviour is nonlinear) is algebraically summated with strain in the elastic state, which is incorrect. In reality [2], $\varepsilon_{a,t} \neq \varepsilon_{a,e} + \varepsilon_{a,p}$. One also uses the Von Mises law for the equivalent state in the plastic domain, which is incorrect as the latter is applicable only to the linear-elastic domain (in general at $\sigma < \sigma_y$).

After carefully analysing the relations used at present in calculating the fatigue loading of a body, it was found the following [2]:

– in the case of material behaviour in the plastic domain, a behaviour known to be nonlinear, one sums up algebraically the elastic and plastic deformations, which is incorrect;

– some laws do not take into account the influence of mean stress, while others force the introduction of the mean stress, either in the left member or in the right member;

– Basquin's law does not contain the influence of the mean stress;

– in the case of modern machinery operating with a high number of cycles, $N > N_0$, fatigue calculation should be reconsidered, as actually, there is no limit stress when $N \geq N_0$;

– the present laws containing the ratio $\sigma_m/\sigma_y$ are usable only in the domain III;

– some laws make use of an unacceptably high number of constants.

*A proposal for solving the problem of fatique loading*

In general, a material behaves differently along different directions and its behaviour is often nonlinear.

The problem that arises and which should be solved is to calculate the total effect of the action of all the forces upon the body, how this total effect is attained and how one can equate these loadings, even if they are different in nature and types.

The solution of these problems of effect superposition and equivalence of different loadings effects can be done by using the principle of critical energy from the Energonics, or V.V. Jinescu principle [1;2]. One considers that the deployment of the phenomena and physical, chemical, physico-chemical processes starts only after an amount of precisely determined specific energy has been absorbed (specific energies are expressed in J/kg or in J/m$^3$).

The amount of energy that triggers the phenomenon or process is named critical specific energy.

If different external factors act upon a mechanical structure, the critical state is attained in certain conditions, under the action of a load group called critical load grouping. The calculation and determination of these conditions has been carried out by using the principle of critical energy which has been stated as follows [1; 9-11]:

„The critical state in a process or phenomenon is reached when the sum of the specific energy amounts involved, considering the sense of their action, becomes equal to the value of the specific critical energy characterizing that particular process or phenomenon".

The mathematical expression of the principle of critical energy for ideal materials (homogeneous, isotropic, free of residual stresses) is:

$$\sum_j E_j \cdot \delta_j = E_{cr}, \tag{16}$$

where $E_j$ is the specific energy involved in the process; $E_{cr}$ is the critical specific energy which is a constant for a given case, independent of the nature or type of energy involved; $\delta_j$ is an undimensional coefficient which depends on the sense of action of the energy involved:

$\delta_j = 1$, if the energy action $E_j$ is in the sense of the phenomenon under consideration;

$\delta_j = 0$, if the energy action $E_j$ has no effect upon the phenomenon under consideration;

$\delta_j$ =-1, if the energy action $E_j$ opposes the phenomenon under consideration.

The expression:

$$P_j = \left(\frac{E}{E_{cr}}\right)_j \cdot \delta_j \quad (17)$$

is the participation of specific energy $E_j$.

Taking into account the relationship (16) one can write [1; 10],

$$P_T(t) = \sum_j P_j, \quad (18)$$

where $P_T$ is the total participation of specific energies, $t$ is the time.

The loading is not dangerous if,

$$P_T(t) < P_{cr}(t), \quad (19)$$

where $P_{cr}(t)$ is the critical participation, $P_{cr}(t) \leq 1$.

By using the principle of critical energy (PCE), there have been solved some cases of superposition of effects in engineering science and in fundamental sciences [10 - 23].

In order to solve some loading cases, one adds the behaviour law to the expression of the principle of critical energy.

One considers the nonlinear behaviour of the material given by the power law [10],

$$Y = C \cdot X^k, \quad (20)$$

where C and k are material constants. In this case relation (17) becomes [1]:

$$P_j = \left(\frac{Y_j}{Y_{j,cr}}\right)^{\alpha+1} \cdot \delta_j, \quad (21)$$

where $\alpha$ =1/k, such as the total participation,

$$P_T(t) = \sum_j \left(\frac{Y_j}{Y_{j,cr}}\right)^{\alpha+1} \cdot \delta_j. \quad (22)$$

Exponent $\alpha$ takes values depending on the mode and speed of applying the external load.

In most cases one does not want the critical state to be attained, but rather maintaining a certain safety distance with respect to it. In this situation, it is introduced the notion of allowable specific energy, $E_a = \frac{E_{cr}}{c_E}$,where $c_E$> 1 is the safety coefficient with respect to specific energy.

Instead of relation (22), we write [2; 10],

$$P_T(t) = \sum \left(\frac{Y_j}{Y_{j,al}}\right)^{\alpha+1} \cdot \delta_j, \quad (23)$$

where $P_T(t)$ is the total participation of specific energy with respect to the allowable state; $Y_{j,al} = Y_{j,cr} / c_y$ is the allowable load; $c_y$ > 1 is the coefficient of safety.

The loading is allowable if,

$$P_T(t) \leq P_{al}, \quad (24)$$

where $P_{al} \leq 1$ is the allowable participation of specific energy.

For ideal materials, free of residual stresses and undeteriorated $P_{al}$ = 1.

For real materials the critical participation is calculated with the general relation [2; 22; 23],

$$P_{cr}(t) = 1 - D_T(t), \quad (25)$$

where $D_T(t)$ is the total deterioration with respect to critical state, a dimensionless variable with the following extreme values: $D_T$ = 0 – for undeteriorated material; $D_T$ = 1 – for totally degraded material (fracture, excessive deformation).

In the paper [22] there have been deduced some expressions for deterioration $D$ in the case of mechanical structures under fatigue load.

The allowable participation of specific energy for real materials is,

$$P_{al} = 1 - D_T^*(t), \quad (26)$$

where $D_T^*(t)$ is the total deterioration with respect to allowable state.

In the case of fatigue loading of structures made of materials with nonlinear behaviour (20), instead of Palmgren-Miner's eq. (3) one recommends V.V. Jinescu's law [24],

$$\sum_j \left(\frac{n}{N}\right)_j^{\frac{\alpha+1}{m}} = C_D, \quad (27)$$

where $\alpha$ =1/$k$, $m$ becomes from the Basquin's law (10) and $C_D$ depends on the mean stress, on the deterioration and on the residual stress. If $N$ is the allowable number of cycles with the stress amplitude $\sigma_{a,j}$, than eq. (27) gives the allowable state of successive cyclic loading.

But, if $N_j$ is the number of cycles at failure with the stress amplitude $\sigma_{a,j}$, then the eq. (27) gives the critical state.

The use of relations (22), (25) and (27) allows the determination of loading conditions that lead to the induction of the critical state in a certain real case. The advantage of these relations lies in the fact that, on the one hand, they do not feature the drawbacks outlined in the previous relations and allow the consideration of the real behaviour of the structure material under study, while, on the other hand, they introduce a term ($D_T$) on whose basis one calculates the total effect of the deteriorations produced upon the mechanical structure.

## Conclusions

Up to the present there have been many methods and laws for material fatigue calculation, but the results obtained after applying some of them, differ, which should not happen in engineering.

Taking into consideration the drawbacks which is elaborated upon in the present paper, one may claim that the law which can be applied to all Wöhler's curve domains and which takes into consideration the superposition of the effects of all the forces upon the respective structure, are the general laws (19) and (24) deduced on the basis of the Principle of critical energy.

This principle, whose applications range widely in all the domains (mechanics, thermodynamics, electrodynamics, electromagnetism, thermodynamics, thermomechanics, chemistry, biology etc), is much closer to what is happening in reality, leading to more accurate, hence more realistic results, than present day relations.

## References


1.JINESCU V.V., Energonica, Editura Semne, Bucureşti, 1997.
2.JINESCU V.V., Tratat de Termomecanică, vol. 1, Editura AGIR, Bucureşti 2011.
3.JINESCU V.V., Rev. Chim. (Bucharest), **59**, no. 4, 2008, p. 453.
4.MINER MA, Cumulative damage in fatigue, J Appl Mech, **67**, 1945, p. A159-A164.
5.PALMGREN A., Die lebensdauervon Kugellagern, Zeitschrift, **68**, 1924, p. 339-410.
6.*** EURONORME EN 13445/2009.
7.BASQUIN OH, The exponent low of endurance tests. Proc Am Soc Test Mater, **10**, 1910, p. 625-630.
8*** ASME Code, Division VIII, Section 2/2009.

9.JINESCU V.V.,Rev. Chim. (Bucharest), **35**, no. 9, 1984, p. 858.
10.JINESCU V.V., Principiul energiei critice şi aplicaţiile sale, Editura Academiei Române, Bucureşti, 2005
11.JINESCUV.V., Principles and Laws of Energonics, Politehnica Press, Bucureşti, 2003.
12.JINESCU V.V., ,Stability Determination of Structures under a group of Loads by using the Principle of Critical Energy, Int. J. Pres.Ves.&Piping, **48**, 1991, p. 343.
13.JINESCU V.V., , Rev. Chim.(Bucharest) , **40**, no. 1, 1989, p. 67; no. 7, 1989, p. 622
14.JINESCU V.V, Rev. Chim. (Bucharest), **40**, no. 8, 1989, p. 677
15. JINESCU V.V., ,The Superposition of Effects Produced by Buckling, Creep and Fatigue, Int. J. Pres. Ves.&Piping, **53**, 1993, p.377-391.
16.JINESCU V. V., Rev. Chim. (Bucharest), **41**, no. 4, 1990, p. 409; **41**, no. 6, 1990, p. 541
17.JINESCU V.V., The Energy Concept in critical Groups of Load Calculation, Int. J. Pres. Ves.&Piping, **38**, nr. 3, 1989, p. 211-226.
18. JINESCU V.V., The Principle of Critical Energy in the Field of Materials Fracture Mechanics, Int. J. Pres. Ves&Piping, **53**, 1993, p. 39-45.
19.JINESCU V.V., Rev. Chim. (Bucharest) **41**, 1990, p. 677, p. 769
20. JINESCU V.V.,Rev. Chim. (Bucharest), **48**, no. 1, 1997, p. 5
21.JINESCU V.V., The High Degree generality of the Principle of Critical Energy, Rev. Roum. Chim., **34**, 1989, p. 491-502.
22.JINESCU V.V., Cumulation of Effects in Calculating the Deterioration of Fatigue Loaded Structures, Int. J. Damage Mechanics, **21**, 2012, p. 671-695.
23.JINESCU V.V., Rev. Chim. (Bucharest)e, **63**, no.1, 2012, p. 98
24.JINESCU V.V., Critical energy approach for the fatigue life calculation under blocks with different normal stresses amplitudes, Int J Mech Sci, **67**, 2013, febr, p. 78-88.

Manuscript received: 10.06.2014