Separation of Heterogeneous Solid – Fluid Mixtures
IX. Filtration theory with new flowing models through packed beds

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Starting from geometrical model based on particles of ideal shape deposed as incompressible precipitate and on extreme degrees of compaction for spherical layers (hexagonal and cubic lattice, respectively), new relations for filtration specific resistances calculus were proposed. The weight between the two extreme layouts, expressed as X_C, gives for the modified traditional model very closed values to the geometrical model. For the engineering approach a model exclusively based on void fraction ε is more convenient, so corrections were suggested. Presented formulas are limited to the constant pressure filtration with incompressible precipitate, consisting from particles of irregular size, but with a calculable shape factor ϕ.

Keywords: filtration theory, filtration modeling, real geometry of solid particles and precipitate pores, specific resistances

In a previous paper [1] two new overall models for laminar flow through layers formed by identical spherical particles were proposed, in two extreme opposite settlings, compact and loose, respectively. The first model is based on the accurate pores' geometry (the flowing cross section and tortuosity) and is further referred as the "geometrical model" while the second one is based on the ambiguous concept of void fraction (porosity, ε), well-known from literature [1÷5], but modified to fit the results of both our first model and experimental data [6] – it will be further referred as the "modified traditional model". Recently, a comparison between results obtained by conventional filtration theory through precipitate and filter material (support) with other approaches of experimental data [9÷14] was done by ***Chi Tien*** and ***Renbi Bai*** [15]. In this paper the extension of these two models for granular beds consisting of non-spherical particles of irregular size, but having volumes (**V**) and surfaces (**A**) which could be calculated using geometrical approximations and a sphericity factor (**ψ**) is presented. In this approach, the degree of compaction for random settling could be estimated either using the mean diameter of particles or from experimental data, taking into account the balance between the two extreme types of layers' order, defined for spherical particles. Based on the adjusted flowing equations, the specific resistances of both precipitate and filter material will be computed to be substituted into the classic form of filtration rate expression.

Flowing models through packed beds composed of non-spherical particles

In a solid – liquid suspension, solid particles are non-spherical even when they arise from processes forming microcrystals with defined geometrical shapes, which yields to incompressible precipitates. According to the previous paper [1], the spherical particles could be imagined sliding from an initial loose layout to a more compact one, by occupying available gaps in the neighborhood; when this compact layout is reached, no further compressing is possible. If the particle geometry could be estimated, using a unique characteristic dimension (***i.e.***, the edge length of a known shape crystal, say **ℓ**) or directly seen (on a microscope), various equivalent diameters, and further the sphericity and the porosity of the bed with random settling could be found (obviously, using more experimental data).

Equivalent diameters and sphericity for non-spherical particles

For different geometrical shapes could be estimated:

- the equivalent diameter of a particle, $\mathbf{d_{ech,p}}$, by matching either the volume to a fictive sphere ($\mathbf{V_p = V_s}$) or the external surface, respectively; it yields:

$$V_p = \frac{\pi \cdot d_V^3}{6} \Rightarrow d_V = \sqrt[3]{\frac{6 \cdot V_p}{\pi}} \quad (1a)$$

$$A_p = \pi \cdot d_A^2 \Rightarrow d_A = \sqrt{\frac{A_p}{\pi}} \quad (1b)$$

- the shape factor (sphericity), expressed in two ways [3, p.186, and 5, p.1373]:

$$\psi = \frac{\pi \cdot d_p^2}{A_p} \text{ or } \phi = \frac{6 \cdot V_p}{d_p \cdot A_p} \quad (2)$$

This becomes, by substituting relations (1):

$$\psi = \frac{\pi \cdot d_V^2}{\pi \cdot d_A^2} = \left(\frac{6 \cdot V_p}{\pi}\right)^{\frac{2}{3}} \cdot \frac{\pi}{A_p} = \sqrt[3]{\pi} \cdot \frac{\left(6 \cdot V_p\right)^{\frac{2}{3}}}{A_p} = \sqrt[3]{\frac{\pi}{6 \cdot V_p}} \cdot \frac{6 \cdot V_p}{A_p} = \frac{6 \cdot V_p}{d_V \cdot A_p} = \phi \quad (3)$$

- the absolute deviation from the spherical form of the particle:

$$\phi \cdot d_V = \frac{6 \cdot V_p}{A_p} = \psi \cdot d_V \quad (4)$$

This quantity will be used further, assuming the calculus of $\mathbf{V_p}$ and $\mathbf{A_p}$ from characteristic dimensions (eventually reduced to only one, **ℓ**). In table 1 results of calculus for $\mathbf{d_V}$, **ψ** and $\boldsymbol{\phi \times d_V/\ell}$ in some usual cases (representative geometrical shapes for some crystallographic classes) are given.

It could be concluded that particles having shapes far beyond from spherical have these values far below unity (positions 4÷7), while the quasi-spherical particles have these values close to the unity (but still below, positions

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Table 1

EQUIVALENT DIAMETERS ($\mathbf{d_V}$) AND SHAPE FACTORS ($\boldsymbol{\psi}$, $\boldsymbol{\phi}$) FOR PARTICLES WITH DEFINITE GEOMETRY

No.	Particle's geometry				Invariant for overall surface $\mathbf{A_p/\ell^2}$	Invariant for volume $\mathbf{V_p/\ell^3}$	$\mathbf{d_V/\ell}$ (1a)	$\boldsymbol{\psi = \phi}$ (3)	$\boldsymbol{\phi \cdot d_V/\ell}$ (4)
	Geometrical shape	Characteristic dimension ℓ	Faces (number and shape)	Number of edges					
1	Sphere	$\ell = d_S$	1_S	–	π	$\pi / 6$	1.0000	1.0000	1.0000
2	Cylinder	$\ell = d_{cil} = h_{cil}$	$2_{cc} + 1_{dr}$	–	$3\pi / 2$	$\pi\sqrt{3} / 12$	1.1447	0.8736	1.0000
3	Cube	$\ell = L_{pat}$	6_{pat}	12	6	1	1.2407	0.8060	1.0000
4	Cone	$\ell = d_{con} = S$	$1_{cc} + 1_{sc}$	–	$3\pi / 4$	$\pi\sqrt{3} / 24$	0.7565	0.7631	0.5774
5	Tetrahedron	$\ell = L_{tech}$	4_{tech}	6	$\sqrt{3}$	$\sqrt{2} / 12$	0.6086	0.6711	0.4083
6	Pyramid	$\ell = L_{tech} = L_{pat}$	$4_{tech} + 1_{pat}$	8	$1+\sqrt{2}$	$\sqrt{2} / 6$	0.7664	0.7643	0.5858
7	Triangular prism	$\ell = L_{tech} = L_{pat}$	$2_{tech} + 3_{pat}$	9	$(6+\sqrt{3}) / 2$	$\sqrt{3} / 4$	0.9386	0.7160	0.6720
8	Rhombic prism	$\ell = L_{tech} = L_{rb}$	$4_{tech} + 2_{rb}$	12	$4 + \sqrt{3}$	$\sqrt{3} / 2$	1.1826	0.7665	0.9065
9	Hexagonal prism	$\ell = 2\ L_{hex} = L_{pat}$	$6_{pat} + 2_{hex}$	18	$(12+3\sqrt{3}) / 4$	$3\sqrt{3} / 8$	1.0745	0.8437	0.9065
10	Octahedron	$\ell = L_{tech}$	8_{tech}	12	$2\sqrt{3}$	$\sqrt{2} / 3$	0.9656	0.8456	0.8164
11	Double cone	$\ell = d_{con} = S$	2_{sc}	–	π	$\pi\sqrt{3} / 12$	0.9532	0.9086	0.8660
12	Icosahedron	$\ell = L_{tech}$	20_{tech}	30	$5\sqrt{3}$	$(15+5\sqrt{5}) / 12$	1.6092	0.9393	1.5115
13	Dodecahedron	$\ell = L_{ptg}$.	12_{ptg}	30	$3(5(5+2\sqrt{5}))^{0.5}$	$5 + 7\sqrt{5} / 3$	2.6922	1.1029	2.9694
14	Average crystal*	$\ell = L_{tech} = L_{pat}$	$8_{tech} + 6_{pat}$	24	$2(3+\sqrt{3})$	$5\sqrt{2} / 3$	1.6512	0.9050	1.4943

*It yields from a cube with all peaks truncated at half of the edges (its circumscribed sphere is tangent to all faces of the initial cube).

8÷11). The particles having the characteristic dimension, **a**, far below the diameter of the circumscribed sphere, have the quantity $\boldsymbol{\phi \times d_V > \ell}$ (positions 12÷14). It should be emphasized that $\boldsymbol{\phi \times d_V}$ will substitute the diameter $\mathbf{d_S}$ in all relations previously established for spherical layers.

Average diameters for irregular size particles

The solid phase of a suspension is generally polydispersed, so a sample of the dried and sieved precipitate is needed to be divided (screened) into $\mathbf{N_g}$ granulometric groups, each one being quasi-monodispersed. Many expressions for the average diameter, $\bar{d}_p$ has been proposed [2-4, 6]. For consistency, the average diameter with respect to the particle volume will be calculated, $\mathbf{d_V}$, with relation (1a). Starting with a sample having the mass **m**, screened into $\mathbf{N_g}$ fractions by sieving, the **j** fraction having the mass $\mathbf{m_j}$ and the diameter $\mathbf{d_j}$ (the fraction being considered monodispersed), and the mass fraction $\mathbf{w_j = m_j / m}$. The mass of a single particle from the **j** fraction is denoted with $\mathbf{m_{pj}}$ and the mass of the average particle from the sample with $\bar{m}_p$. The number of particles from **j** fraction will be $\mathbf{n_j}$, respectively, for the entire sample,

$$N_p = \sum_{j=1}^{N_g} n_j$$

It yields:

$$m = \bar{m}_p \cdot N_p = \bar{m}_p \cdot \sum_{j=1}^{N_g} n_j = \bar{m}_p \cdot \sum_{j=1}^{N_g} \frac{m_j}{m_{pj}} = m \cdot \bar{m}_p \cdot \sum_{j=1}^{N_g} \frac{w_j}{m_{pj}} \quad \Rightarrow \quad \bar{m}_p = \left(\sum_{j=1}^{N_g} \frac{w_j}{m_{pj}} \right)^{-1}$$

where:

$$\bar{m}_p = \rho_p \cdot \bar{V}_p = \rho_p \cdot \frac{\pi \cdot d_V^3}{6} = \left(\sum_{j=1}^{N_g} \frac{6}{\pi \cdot \rho_p} \cdot \frac{w_j}{d_j^3} \right)^{-1}$$

and, using (1a), it follows:

$$d_V = \left(\sum_{j=1}^{N_g} \frac{w_j}{d_j^3} \right)^{-1/3} = \sqrt[3]{\frac{6 \cdot \bar{V}_p}{\pi}} \qquad (5)$$

Finally, after $\mathbf{d_V}$ calculus with (5) using $\mathbf{N_g}$ diameters $\mathbf{d_j}$, several well-defined particles (as shape) from the fraction having the closest diameter to $\mathbf{d_V}$ and it yields, with $\bar{V}$ and $\bar{A}$, quantities $\mathbf{d_V}$ and $\boldsymbol{\phi}$, the product $\boldsymbol{\phi \times d_V}$ being used further to characterize the behaviour of all particles from the granular bed, corresponding to the diameter $\mathbf{d_S}$ in case of a granular bed having the same size and the same spherical shape (***i.e.***, with sphericity 1).

Void fraction ε, degree of compaction X_C (for the precipitate layer), tortuosity factor k_{tt}

For spherical particles of the same size and a uniform settling in the granular bed, $\boldsymbol{\varepsilon}$ is not a function of the

diameter $\mathbf{d_s}$. The degree of compaction, defined as a ratio and denoted with $\mathbf{X_c}$, lies between the two extreme dispositions (limits) [1]: compact ($\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon_c} = 25.95\%$), corresponding to $\mathbf{X_c} = 1$, and loose, respectively ($\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon_a} = 47.64\%$), corresponding to $\mathbf{X_c} = 0$:

$$X_C = \frac{\varepsilon_a - \varepsilon}{\varepsilon_a - \varepsilon_C} = \frac{6 - (\pi + 6 \cdot \varepsilon)}{\pi \cdot (\sqrt{2} - 1)} = 2.1966 - 4.6108 \cdot \varepsilon; \quad (0.2595 < \varepsilon < 0.4764) \tag{6}$$

Using the ratio $\mathbf{X_c}$ could be defined also a tortuosity factor $\mathbf{k_{tt}}$ [1], bounded by 1 and $\mathbf{k_{tt,C}}$ respectively, with expression:

$$k_{tt} = k_{tt,C} \cdot X_C + k_{tt,a} \cdot (1 - X_C) = \left[\pi \cdot \left(\sqrt{2} - 1\right) - 1\right] \cdot X_C + 1 = 0.3013 \cdot X_C + 1 \tag{7}$$

This corresponds to:

$$\varepsilon = \varepsilon_C \quad \Leftrightarrow \quad k_{tt,C} = 1.3013 \text{ and } \varepsilon = \varepsilon_a \quad \Leftrightarrow \quad k_{tt,a} = 1$$

so the weighted pore length in a layer with a given thickness $\boldsymbol{\delta}$:

$$\ell_{por} = k_{tt} \cdot \delta = \left[\left[\pi \cdot \left(\sqrt{2} - 1\right) - 1\right] \cdot X_C + 1\right] \cdot \delta \tag{8}$$

It follows:

$$\varepsilon = \varepsilon_C \quad \Rightarrow \quad \ell_{por} = \ell_C = 1.3013 \cdot \delta \text{ and}$$
$$\varepsilon = \varepsilon_a \quad \Rightarrow \quad \ell_{por} = \ell_a = \delta$$

For any intermediate value $\boldsymbol{\varepsilon_c < \varepsilon < \varepsilon_a}$, which corresponds to a degree of compaction $\mathbf{X_c}$, it yields (in case of spherical particles layer) $\boldsymbol{\delta < \ell_{por} < \ell_c}$.

For the dependence $\boldsymbol{\varepsilon(d_V)}$, experimental data available in literature are so poor that no confident conclusions could be derived. Only a vague assertion that, for a given geometry, increasing the particles' dimensions yields to a decrease of the specific surface $\boldsymbol{\sigma}$ and also to an increase value for $\boldsymbol{\varepsilon}$ was done [3]. If the first part is obvious, the second one is justified only by intuition. From data presented in tables 2 and 3 it follows that, even for spherical granular beds quasi-monodispersed, there is no certain prediction for evolution of dependencies $\boldsymbol{\varepsilon(d_s)}$ or $\boldsymbol{X_c(d_s)}$.

From table 1, for layers containing spheres with diameters below 1 mm, $\boldsymbol{\varepsilon(d_s)}$ is monotone decreasing and also $\boldsymbol{\sigma_c(d_s)}$, resulting from specific surface expression:

$$\sigma_C = \sigma_S \cdot (1 - \varepsilon) = \frac{6}{\phi \cdot d_V} \cdot (1 - \varepsilon) \tag{9}$$

This assertion disagrees with table 2; according to data above the degree of compaction $\mathbf{X_c}$ increase slowly with $\mathbf{d_s}$. Although non-expected, for non-conductive little particles, this tendency could be explained – when they are free poured in air – by electrical repulsive charging. This follow to loose granular beds while the settling with 12 contact points to other particles (compact) is switched – at least partially – to only 6 contact points (***i.e.***, by changing the hexagonal lattice to a cubic one). It could be stated that with decreasing of the diameter $\mathbf{d_s}$ this tendency is much pronounced.

From table 3, given data (with unspecified source) for spherical particles with $\mathbf{d_s} > 1$ mm indicate a slow and non-uniform increasing of the dependence $\boldsymbol{\varepsilon(d_s)}$, but

No.	$\mathbf{d_S} \times 10^4$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\sigma_C} \times 10^{-3}$ (9)	$\mathbf{X_C}$ (6)	$\mathbf{k_{tt}}$ (7)
	m	–	m^{-1}	–	–
1	0.55	0.454	59.56	0.1033	1.031
2	0.72	0.441	46.58	0.1632	1.049
3	0.89	0.434	38.16	0.1955	1.059
4	1.01	0.424	34.22	0.241	1.073
5	2.07	0.411	17.07	0.3016	1.091
6	2.84	0.39	12.89	0.3984	1.12
7	5.51	0.385	6.7	0.4214	1.127
8	28.9	0.386	1.27	0.4168	1.126

*Neither material nor granulometric distribution specified.

Table 2
LAYER OF BULK SPHERES* – $\boldsymbol{\varepsilon}$ AFTER **GELDART** [7]

No.	$\mathbf{d_S} \times 10^3$	$\boldsymbol{\sigma}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon_{calc}}$ (9)*	$\mathbf{X_C}$ (6)*	$\mathbf{k_{tt}}$ (7)*
	m	m^{-1}	–	–	–	–
1	2	2100	0.300	–	0.813	1.245
2	6 ÷ 7	400	0.400	0.567	0.352	1.110
3	9 ÷ 10	315	0.405	0.501	0.329	1.099
4	12 ÷ 13	235	0.410	0.510	0.306	1.092
5	15 ÷ 16	210	0.410	0.458	0.306	1.092
6	19 ÷ 20	157	0.410	0.490	0.306	1.092
7	25 ÷ 26	125	0.418	0.469	0.269	1.081
8	35 ÷ 37	85	0.446	0.490	0.140	1.042
9	50 ÷ 52	65	0.450	0.448	0.122	1.036

*calculus with average values for diameter $\mathbf{d_S}$ and relations (9, 6, 7); hydrodynamic found $\boldsymbol{\varepsilon}$ [5].

Table 3
LAYER OF BULK CERAMIC SPHERES [3]

$\varepsilon_{calc}(d_S)$ as a result of specific surface data (σ) show a decreasing with come-backs, so a pertinent conclusion is impossible (for $\varepsilon > \varepsilon_a$, $X_C < 0$). Only extreme diameters present some confidence; perhaps beds vibrating, eventually sub-merged in liquid, and using frequencies according to particles' size, could yield to much accurate and reproducible data, and easier to interpret.

Extension of above results to filtration (retention of solid particles from suspension) is difficult, because of their variable size and shape. In this case, the hydrodynamic method for estimating the quantity ε is recommended [5]. One could anticipate that, for particles of different size, the smallest depose by filtration inside the gaps left by the larger ones in the precipitate, so a decreasing of ε(and an increasing of X_C) is expected during the time, since the thickness of the layer (δ) is growing, together with capillary length (ℓ) and their tortuosity (k_{tt}).

Adjusted relations for non-spherical particles

Inside the two proposed models for the laminar flow through granular beds containing only identical spherical particles [1], some adjustments are required for layers containing non-spherical particles with different sizes, which yield to a sort of disorder inside the bed; previously given relations should be corrected, starting from known values for d_i and w_i, continuing with average values d_V from (5), then ε (experimental) and further sphericity ϕ, and finally the product $\phi \times d_V$ given by (4).

The geometrical model [1] will be modified for a certain degree of compaction $X_C(\varepsilon)$ from (6) and a tortuosity factor k_{tt} from (7), calculated with ε(fig. 1), as follows:

- the capillary average diameter, d_{por}, is obtained by weighting with X_C the two extreme diameters, d_C and d_a, corresponding to compact and loose settling, respectively:

$$d_{por} = X_C \cdot d_C + (1 - X_C) \cdot d_a, \text{ with}$$
$$d_C = \frac{2 \cdot \sqrt{3} - 3}{3} \cdot \phi \cdot d_V \text{ and } d_a = \left(\sqrt{2} - 1\right) \cdot \phi \cdot d_V \quad (10)$$

Obviously, ε from X_C should satisfy the condition $\varepsilon_C < \varepsilon < \varepsilon_a$. For small and quasi-spherical particles ($d_V < 1$ mm), this condition is always satisfied. The diameter d_S from all flowing relations should be replaced with the product $\phi \times d_V$ given by (4), where $\overline{V}_p$ and $\overline{A}_p$ are averages for the size and shape of particles, found by experiment (eventually microscopic);

- the average flow rate through pores having the length $\ell = k_{tt} \cdot \delta$ and the diameter d_{por} yields from ***Hagen – Poiseuille*** relation:

$$v = \frac{\Delta p_{tot} \cdot d_{por}^2}{32 \cdot \mu \cdot k_{tt} \cdot \delta} = \frac{\Delta p_{tot}}{32 \cdot \mu \cdot \delta} \cdot \frac{\left[X_C \cdot d_C + (1 - X_C) \cdot d_a\right]^2}{X_C \cdot k_{tt,C} + (1 - X_C) \cdot k_{tt,a}} \quad (11)$$

with $\Delta p_{tot} = p_0 - p_\ell \pm h \cdot \rho \cdot g$ (the **h** sign is given by gravity); $\delta = \delta(t)$ is the time-variable (**t**) thickness of the precipitate;

$$k_{tt,C} = \pi \cdot \left(\sqrt{2} - 1\right), \; k_{tt,a} = 1.$$

- the pore number, N_{por}; if they are approximated to cylinders, having all the same diameter d_{por} and being regular distributed on the filtration area A_f, which is perpendicular on the flowing direction:

$$N_{por} = N_C \cdot X_C + N_a \cdot (1 - X_C) = \left[\frac{4 \cdot \sqrt{3}}{3} \cdot X_C + (1 - X_C)\right] \cdot \frac{A_f}{(\phi \cdot d_V)^2} \quad (12)$$

- the ***Reynolds*** number and the laminar condition:

$$\mathrm{Re}_{por} = \frac{v \cdot d_{por} \cdot \rho}{\mu} = \frac{\Delta p_{tot} \cdot \rho}{32 \cdot \mu^2 \cdot \delta} \cdot \frac{\left[X_C \cdot d_C + (1 - X_C) \cdot d_a\right]^3}{X_C \cdot k_{tt,C} + (1 - X_C) \cdot k_{tt,a}} < 2300 \quad (13)$$

- the volumetric flow G_V on the entire filter area, A_f:

$$G_V = G_{V,C} \cdot X_C + G_{V,a} \cdot (1 - X_C) = v \cdot A_{por} \cdot N_{por} =$$
$$= \frac{\Delta p_{tot} \cdot d_{por}^2}{32 \cdot \mu \cdot k_{tt} \cdot \delta} \cdot \frac{\pi \cdot d_{por}^2}{4} \cdot \left[\frac{4 \cdot \sqrt{3}}{3} \cdot X_C + (1 - X_C)\right] \cdot \frac{A_f}{(\phi \cdot d_V)^2} =$$
$$= \frac{\pi \cdot \Delta p_{tot} \cdot d_{por}^4}{128 \cdot \mu \cdot \ell_{por}} \cdot \left[\frac{4 \cdot \sqrt{3}}{3} \cdot X_C + (1 - X_C)\right] \cdot \frac{A_f}{(\phi \cdot d_V)^2} \quad (14)$$

The so-called modified traditional model (using the void fraction ε) [1] will be altered as shown:

- the void fraction (weighted):

$$\varepsilon = \frac{V_{strat} - V_p}{V_{strat}} = X_C \cdot \varepsilon_C + (1 - X_C) \cdot \varepsilon_a \quad (15)$$

- the modified equivalent diameter, for the gaps:

$$d_g = \frac{2}{3} \cdot d_{ech} = \frac{2}{3} \cdot \left(\frac{2}{3} \cdot \frac{\varepsilon}{1 - \varepsilon}\right) \cdot \phi \cdot d_V = \left(\frac{2}{3}\right)^2 \cdot \frac{\varepsilon}{1 - \varepsilon} \cdot \phi \cdot d_V \quad (16)$$

which could be assimilated with $d_g = X_C \cdot d_{gC} + (1\text{-}X_C) \cdot d_{ga}$, where d_{gC} and d_{ga} refer to the spherical particle's layer ($\phi \cdot d_V = d_S$) – this assimilation remains for next quantities;

- the average flow rate inside the gaps:

$$v_g = \frac{\Delta p_{tot} \cdot d_g^2}{32 \cdot \mu \cdot \ell_{por}} = \frac{\Delta p_{tot}}{32 \cdot \mu \cdot k_{tt} \cdot \delta} \cdot \left(\frac{2}{3}\right)^4 \cdot \left(\frac{\varepsilon}{1 - \varepsilon}\right)^2 \cdot (\phi \cdot d_V)^2 \quad (17)$$

- the Reynolds number and the laminar condition:

$$\mathrm{Re}_g = \frac{v_g \cdot d_g \cdot \rho}{\mu} = \frac{\Delta p_{tot}}{32 \cdot \mu^2 \cdot k_{tt} \cdot \delta} \cdot \left(\frac{2}{3}\right)^6 \cdot \left(\frac{\varepsilon}{1 - \varepsilon}\right)^3 \cdot (\phi \cdot d_V)^3 < 2300 \quad (18)$$

- the volumetric flow on the entire filter area, A_f:

$$G_{V,g} = \frac{2}{3} \cdot \varepsilon^2 \cdot v_g \cdot A_f = v_0 \cdot A_f \quad (19a)$$

where:

$$v_0 = \frac{2}{3} \cdot \varepsilon^2 \cdot v_g = \frac{\Delta p_{tot}}{32 \cdot \mu \cdot k_{tt} \cdot \delta} \cdot \left(\frac{2}{3}\right)^5 \cdot \left(\frac{\varepsilon^2}{1 - \varepsilon}\right)^2 \cdot (\phi \cdot d_V)^2$$

is the superficial (fictive) velocity, related to the area A_f.

It was shown that both methods give more or less equivalent results, at least for identical spherical small particles ($d_S < 1$ mm), with a deviation of maximum 7.5% [1], the first one being more rational (stated as a reference), while the last one is easier to apply.

Filtration theory through incompressible precipitate and filter medium

Filtration, as a separation process of heterogeneous fluid – solid mixtures, has two major aspects (unit operations): de-dusting, when the fluid is a gas, and the properly filtration, when the fluid is a liquid, respectively. The following assertions are available only for the last case.

The main variables in filtration processes are the velocity (**v**) or the filtrate volumetric flow (G_V), and the pressure drop (Δp_{tot}), which is responsible for the fluid flowing through pores of already settled precipitate and of the filter medium at the end. While the thickness of the precipitate (and consequently the pore length) increases during the

time of the process **t**, filtration could occur at constant pressure drop Δp_{tot} = constant (usually), respectively at $v(G_v)$ = constant, when the pressure drop should grow with time, so there is a dependence $\Delta p_{tot}(t)$. Although filtration is a discontinuous process in depth, as an unit operation it could be realized discontinuous (gravitational), pseudo-continuous (with constant feed of suspension in press-filters), or even quasi-continuous (having several successive stages, like filtration, washing, squeezing, discharging, reconditioning), in cellular drum filters or band filters.

Using the constant pressure drop technique, the filtration speed could be increased via:

- increasing the particles' diameter (d_p), using flocculants and/or coagulant additives;
- increasing the filtrate fluidity, i.e., decreasing the dynamic viscosity μ, by suspension heating;
- increasing the pressure difference Δp_{tot}, for incompressible precipitates by increasing the liquid column for gravitational filtration, by increasing the pressure in the suspension room and/or by decreasing the pressure in the filtrate room (this procedure being limited by the vapor pressure of the liquid), by increasing the diameter and/or the rotative speed of the centrifuge etc.

Filtration at Δp_{tot} = constant with deposition of incompressible precipitate, using the geometrical model

Filtration at a constant temperature of a suspension composed of a Newtonian fluid (μ=constant, ρ= constant), is considered. It produce an incompressible precipitate, having solid particles with definite forms, non-deformable, characterized by ρ_S, **f**, d_v; they are deposed on a support (canvas, porous plate, metallic profiles) with d_0 – the pore diameter and δ_0 – the pore length, equal with the thickness of the filter layer, both constant during the process. Because in most cases 1 μm < d_v < 1 mm and consequently the precipitate's pores diameter 0.1 μm < **d** < 0.1 mm, it follows that flowing regime within capillary is laminar. The filter layer with δ_0 = constant, and with an almost constant ε_0 (in first few moments, a partial clogging occurs because of the little particles; during this period, the filtrate is turbid and need to be recycled) yields to a constant resistance $R_{f,0}$. In the same time, the precipitate thickness is variable $\delta(t)$, having also a variable repartition of pressure drop Δp_{tot} (fig. 1). The overall difference Δp_{tot} is maintained constant, but the intermediate value p_i decreases fast in the beginning of filtration from $p = p_0$ to a value which remains almost constant after a period since the filter resistance doesn't changes further, so $\Delta p_o = p_i - p_l$ tends fast to a constant value.

Another necessary quantity for the calculus is the porosity ε, which could be assumed constant if the filter is

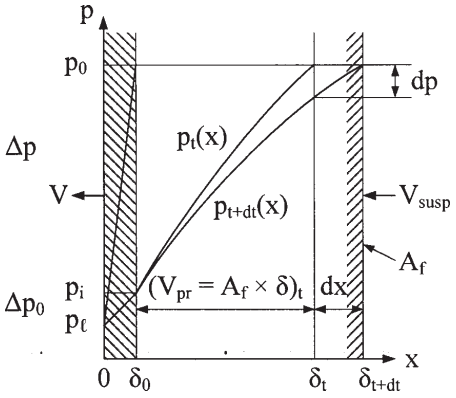

Fig.1. Variation of $p(t,x)$ in support (0) and precipitate (p_t)

$$\Delta p_{tot} = \Delta p_0 + \Delta p = p_i - p_\ell + p_0 - p_i = p_0 - p_\ell = \text{constant}$$

continuously feed with suspension having an uniform distribution of the solid (by mixing in rotative filters, by pumping in press filters etc.). The void fraction ε could be found only by experiment (taking an assay from the suspension, filtering in – almost – same conditions, and drying the precipitate): after washing the precipitate and eventually replacing the retained liquid with a more volatile one (***e.g.***, after washing with water, replacing could be done with methanol), by weighing the wet precipitate (still having a very thin but visible liquid film), and afterwards the dried one, the difference could yield (knowing the density of vaporized liquid) to the void volume and further to void fraction ε by division. Taking into account the trend of the solid particles to settle against the pores' opening, an advanced compaction could be expected and conditions $\varepsilon_c < \varepsilon < \varepsilon_a$ (or $0 < X_c < 1$) are obeyed.

According to figure 1 and considering that $\Delta p = \Delta p_{tot} - \Delta p_o = p_o - p_i$ has only a small variation (***i.e.***, p_i varies during a short period Δt), the pressure on the whole layer $p(x)$ is variable because of $\delta(t)$ during the entire filtration time. The pressure gradient on **x** direction against the flowing sense of the filtrate is positive and could be estimated from ***Hagen – Poiseuille*** equation, written for a current value of **t**:

$$\frac{dp}{dx} = \frac{32 \cdot v \cdot \mu \cdot k_{tt}}{d^2} \qquad (20)$$

where the velocity **v**, being independent on **t**, could be found from (11) and should be used in (14) to calculate the momentary volumetric flow $G_v = d_v / d_t$ related to entire filter surface A_f, **dV** being the infinitesimal variation of filtrate volume from **t** till **t + dt** (corresponding with growing of precipitate thickness δ, from **x** to **x + dx**). The filtrate volume **V** obtained in time **t**, with respect to the filter surface A_f, passing through the entire precipitate thickness δ, is the same for all elementary layers, including the **dx** one, located at the precipitate surface, in contact with suspension. To calculate the solid quantity at time **t** in the precipitate, m_{solid}, **dx** should be replaced in relation (20), knowing the elementary volume $dV_{pr} = A_f \cdot (1-\varepsilon) \cdot dx$, while the elementary mass is $dm_{solid} = \rho_{solid} \cdot (1-\varepsilon) \cdot A_f \cdot dx$, with δ_{olid} being the density of the deposit. By integration one could get:

$$\int_{p_i}^{p_0} dp = \frac{32 \cdot v \cdot \mu \cdot k_{tt}}{\rho_{solid} \cdot A_f \cdot (1-\varepsilon) \cdot d^2} \cdot \int_0^{m_{solid}} dm_{solid} \Rightarrow$$

$$\Delta p = p_0 - p_i = \frac{32 \cdot v \cdot \mu \cdot k_{tt}}{\rho_{solid} \cdot A_f \cdot (1-\varepsilon) \cdot d^2} \cdot m_{solid} \qquad (21)$$

Because the entire solid phase quantity, m_{solid}, remains in the precipitate, m_{pr}, from mass balance it yields:

$$m_{solid} = V_{solid} \cdot \rho_{solid} = V_{pr} \cdot (1-\varepsilon) \cdot \rho_{solid} = m_{susp} \cdot w_{susp} \qquad (22)$$

where w_{susp} is the solid mass fraction in suspension (measurable). The overall suspension mass, m_{susp}, could be obtained by adding the quantity passing into filtrate, $V \cdot \rho$, with the one remaining in the wet precipitate, $V_{pr} \cdot \varepsilon \cdot \rho$:

$$m_{susp} = V_{pr} \cdot (1-\varepsilon) \cdot \rho_{solid} + V_{pr} \cdot \varepsilon \cdot \rho + V \cdot \rho \qquad (23)$$

From relations (22) and (23) it yields:

$$V_{pr} \cdot \left[(1-\varepsilon) \cdot \rho_{solid} \cdot (1-w_{susp}) - \varepsilon \cdot \rho \cdot w_{susp} \right] = V \cdot \rho \cdot w_{susp} \qquad (24)$$

and further:

$$\frac{V_{pr}}{V}=\frac{\rho\cdot w_{susp}}{(1-\varepsilon)\cdot\rho_{solid}\cdot(1-w_{susp})-\varepsilon\cdot\rho\cdot w_{susp}}=\frac{A_f\cdot\delta}{V}=\chi \quad (25)$$

One could assume that above ratio denoted with $\boldsymbol{\chi}$ is invariant with duration, so is the same at the end, if the suspension is well mixed (concentration $\mathbf{w_{susp}}$ is constant in time and uniform in space). From (25):

$$\delta=\chi\cdot\frac{V}{A_f} \quad (26)$$

Filtration equation for incompressible precipitates derives from relation (14) for the volumetric flow $\mathbf{G_V}$. A superficial velocity $\mathbf{v_0}$, related to the entire filter surface $\mathbf{A_f}$, is defined: for the variable thickness $\boldsymbol{\delta}$:

$$v_0=\frac{G_V}{A_f}=\frac{\Delta p\cdot d^2}{32\cdot\mu\cdot k_{tt}\cdot\delta}\cdot\frac{\pi\cdot d^2}{(\phi\cdot d_V)^2}\cdot\left[\left(\frac{\sqrt{3}}{3}-\frac{1}{4}\right)\cdot X_C+\frac{1}{4}\right]=\frac{1}{A_f}\cdot\frac{dV}{dt} \quad (27)$$

It follows:

$$G_V=\frac{dV}{dt}=\frac{A_f\cdot\Delta p}{\frac{32\cdot k_{tt}}{\pi\cdot\left[\left(\frac{\sqrt{3}}{3}-\frac{1}{4}\right)\cdot X_C+\frac{1}{4}\right]}\cdot\frac{(\phi\cdot d_V)^2}{d^4}\cdot\mu\cdot\delta}=\frac{A_f\cdot\Delta p}{r\cdot\mu\cdot\chi\cdot\frac{V}{A_f}} \quad (28a)$$

The same momentary flow passes through the support (the filter made from natural or synthetic canvas, metallic net, sintering glass or ceramic porous plates); by analogy:

$$G_V=\frac{dV}{dt}=\frac{A_f\cdot\Delta p_0}{\frac{32\cdot k_{tt0}}{\pi\cdot\left[\left(\frac{\sqrt{3}}{3}-\frac{1}{4}\right)\cdot X_{C0}+\frac{1}{4}\right]}\cdot\frac{(\phi\cdot d_V)_0^2}{d_0^4}\cdot\mu\cdot\delta_0}=\frac{A_f\cdot\Delta p_0}{r_0\cdot\mu} \quad (29)$$

Replacing $\Delta p_{tot}=\Delta p+\Delta p_0$ and assuming the flow $\mathbf{G_V}$ given by relation (14) passes through both resistances, of the variable precipitate $\boldsymbol{\delta}$, from equation (26), and of the constant support:

$$G_V=\frac{dV}{dt}=\frac{\Delta p_{tot}}{r\cdot\mu\cdot\chi\cdot\frac{V}{A_f^2}+r_0\cdot\mu\cdot\frac{1}{A_f}} \quad (30)$$

In relations (28÷30) were denoted with $\mathbf{r}$, and $\mathbf{r_0}$ respectively, specific resistances of the precipitate and support (the last one includes the constants thickness $\boldsymbol{\delta_0}$). Switching to mass flow $G_m=G_V\cdot\rho$, which divided by $\mathbf{A_f}$, represents the mass flux:

$$q_m=\frac{G_m}{A_f}=\rho\cdot\frac{dV}{A_f\cdot dt}=\frac{\Delta p_{tot}}{R_f+R_{f0}}=\frac{\Delta p_{tot}}{R_{f,tot}} \quad (31)$$

where $R_f=\frac{r\cdot\mu\cdot\delta}{\rho}$ is the internal resistance, against the filtrate flowing through the precipitate with variable $\boldsymbol{\delta}$, while $R_{f,0}=\frac{r_0\cdot\mu}{\rho}$ is the external resistance, of the support. Both have velocity dimensions (m/s), are given by the viscosity, $\boldsymbol{\mu}$, of the filtrate, and they are against the filtrate flow. Relation (31) is very much alike the generalized ***Ohm*** law – a property flux (here the mass flux $\mathbf{q_m}$, similar to the current density $\mathbf{q_{el}=I/A}$, in A/m^2) is expressed like a ratio between a potential gradient which produce the transport (here of pressure, ∇p, analog with the voltage, ∇U), and a resistance to the transport (here $\mathbf{R_f/\ell}$, analog with electrical specific resistance $\boldsymbol{\rho_{el}}$). It follows that filtration flow is somehow analogous with the amperage in a circuit fed by a consumable battery (having a growing electrical resistance). From equation (29) one could find that specific filtration resistances depend on pores' diameter ($\mathbf{d^2}$). By rearranging and integration between obvious limits (for $\mathbf{t}$, from 0 till $\mathbf{t_f}$, and for $\mathbf{V}$, between 0 and $\mathbf{V_f}$) it yields:

$$\int_0^{V_f}\left(\frac{r\cdot\mu\cdot\chi}{A_f^2\cdot\Delta p_{tot}}\cdot V+\frac{r_0\cdot\mu}{A_f\cdot\Delta p_{tot}}\right)\cdot dV=\int_0^{t_f}dt\ \Rightarrow\ a\cdot V_f^2+b\cdot V_f=t_f \quad (32)$$

which is the ***Guth*** relation for filtration with incompressible precipitate, denoting with

$$a=\frac{r\cdot\mu\cdot\chi}{2\cdot A_f^2\cdot\Delta p_{tot}}\ \left(\frac{s}{m^6}\right)\ \text{and}\ b=\frac{r_0\cdot\mu}{A_f\cdot\Delta p_{tot}}\ \left(\frac{s}{m^3}\right)$$

respectively. Both constants are usually found from experimental data linear fitting with relation (32) – precisely, quantities $\mathbf{r}$ and $\mathbf{r_0}$, then using relations (28) and (29), where only size and shape of particles are implied ($\boldsymbol{\phi}$, $\mathbf{d_V}$) and layers characteristics ($\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon_0}$). All these quantities are involved in calculus of filtration constants $\mathbf{a}$ and $\mathbf{b}$ on industrial scale.

Filtration equation using the modified traditional model

By far, using the overall quantities like $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$, defining the granular bed, is more attractive, but these dimensions could not describe the real flow through. It was shown [1] that not the whole section of pores participates to the filtrate flow (at least for layers composed of identical spherical particles), so a realistic geometrical model should take into account stagnant zones (***i.e.***, the void section $\boldsymbol{\varepsilon\cdot A_f}$ is not entirely available for flowing). This assumption should modify the equivalent diameter $\mathbf{d_{ech}}$ with relation (16) for the "gaps" diameter, $\mathbf{d_g}$, which alters significant all flowing relations (17÷19) so much closer results to the geometrical model could be obtained, without any other corrections to fit experimental data. In usual methods for resistances $\mathbf{r}$ and $\mathbf{r_0}$ calculus various numerical (and yet, empirical) coefficients for pressure drops in granular beds at a given velocity are present. For example, in *Kozeny – Carman* [4] relation, for a spherical layer:

$$\Delta p=180\cdot\frac{\delta}{d_S^2}\cdot\left(\frac{1-\varepsilon}{\varepsilon}\right)^2\cdot v\cdot\mu=180\cdot\frac{\delta}{d_S^2}\cdot\frac{(1-\varepsilon)^2}{\varepsilon^3}\cdot v_0\cdot\mu \quad (33)$$

the coefficient 180 (***i.e.***, 72 × 5/2) is nothing else but the theoretical coefficient of 72 from the *Poiseuille* relation, adjusted with the factor of 5/2. In the best known relation for laminar flow through granular beds, *Ergun* [8] suggest the correction factor of 25/12, which lead to a coefficient of 150. The definition of voids equivalent diameter for the granular bed (if only spherical particles are present), after *Kozeny*, is:

$$d_{ech}=\frac{4\cdot\varepsilon}{\sigma}=\frac{2}{3}\cdot\frac{\varepsilon}{1-\varepsilon}\cdot d_S \quad (34)$$

Since $\mathbf{d_{ech}}$ is present in the denominator of (33) as a square (replacing $\mathbf{d_S}$), it yields the value of 72 for the factor (32 × 9/4). It follows that experimental data claim a correction of this coefficient, at least with a factor included between the two values (2.5 respectively 2.083). In other sources [4] even the value 2.78 could be reached. It could be concluded that definition (34) doesn't represent the real flowing section through granular beds voids. An alternative definition for the diameter was introduced with relation (15), which drives the correction coefficient to 2.25, an

average between traditional models. Furthermore, the geometric model imposes for the effective free section of flowing the expression $\mathbf{2/3\ \varepsilon \cdot A_f}$ (19) instead of $\mathbf{\varepsilon \cdot A_f}$ so it's obvious that the current manner of estimating the flowing section gives always bigger values (because of pellicle stagnant zones).

From all above assertions, one could conclude that specific resistances are much closer to reality if they are calculated with relations involving $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma_S = 6 / d_V}$:

$$r = \frac{5 \cdot \sigma_S^2 \cdot (1-\varepsilon)^2}{\varepsilon^3}; \quad r_0 = \frac{5 \cdot \sigma_{S0}^2 \cdot (1-\varepsilon_0)^2}{\varepsilon_0^3}$$

in which specific surface $\boldsymbol{\sigma}$ for non-spherical layers (eventually non-uniform also), could be found from relation (9), written accordingly for them. In these expressions, the numerical factor of 5 comes from $180 / 6^2$ (with 6 from definition of $\boldsymbol{\delta_s}$). The modified model allows the definition of both specific resistances replacing directly the effective diameter of gaps, $\mathbf{d_g}$ from (16), in expressions (29) for $\mathbf{G_V}$ and (19) for $\mathbf{G_{Vg}}$.

Proposed models, coming from substitution of capillary diameters $\mathbf{d}$ (and $\mathbf{d_0}$) for the geometrical model, and $\mathbf{d_g}$ ($\mathbf{d_{g0}}$) for the modified model, could be written:

$$G_V = v \cdot N_{por} \cdot A_{por} = \frac{\Delta p \cdot d^2}{32 \cdot \mu \cdot \ell_{por}} \cdot N_{por} \cdot \frac{\pi \cdot d^2}{4} =$$

$$= \frac{\Delta p \cdot A_f}{32 \cdot \mu \cdot \delta \cdot \left[\pi \cdot (\sqrt{2}-1) \cdot X_C + 1 - X_C\right]} \qquad (28b)$$

$$\frac{\pi}{4} \cdot \left(\frac{4\sqrt{3}}{3} \cdot X_C + 1 - X_C\right) \cdot \left[\left(\frac{2\sqrt{3}}{3} - 1\right) \cdot X_C + (\sqrt{2}-1) \cdot (1-X_C)\right]^4 \cdot (\phi \cdot d_V)^2$$

respectively:

$$G_{V,g} = v_g \cdot \left(\frac{2}{3} \cdot \varepsilon^2 \cdot A_f\right) = \frac{\Delta p \cdot A_f}{32 \cdot \mu \cdot \delta \cdot \left[\pi \cdot (\sqrt{2}-1) \cdot X_C + 1 - X_C\right]} \qquad (19b)$$

$$\left(\frac{2}{3}\right)^5 \cdot \left(\frac{\varepsilon^2}{1-\varepsilon}\right)^2 \cdot (\phi \cdot d_V)^2$$

It follows the calculus definition for both specific resistances $\mathbf{r}$ and $\mathbf{r_0}$, for precipitate and support, each one of them in two ways:

- for the precipitate layer, using relations (28b) and (19b):

$$r = \frac{32}{\frac{\pi}{4} \cdot \left(\frac{4\sqrt{3}}{3} \cdot X_C + 1 - X_C\right)} \cdot \frac{\pi \cdot (\sqrt{2}-1) \cdot X_C + 1 - X_C}{\left[\left(\frac{2\sqrt{3}}{3} - 1\right) \cdot X_C + (\sqrt{2}-1) \cdot (1-X_C)\right]^4 \cdot (\phi \cdot d_V)^2} \qquad (35)$$

$$r_g = \frac{32 \cdot k_{tt}}{\left(\frac{2}{3}\right)^5 \cdot \left(\frac{\varepsilon^2}{1-\varepsilon}\right)^2 \cdot (\phi \cdot d_V)^2} = \frac{32 \cdot \left[\pi \cdot (\sqrt{2}-1) \cdot X_C + 1 - X_C\right]}{\left(\frac{2}{3}\right)^5 \cdot \left(\frac{\varepsilon^2}{1-\varepsilon}\right)^2 \cdot (\phi \cdot d_V)^2} \qquad (36)$$

- for the support, giving identical expressions, but with the subscript (0), characteristic for this layer ($\mathbf{r_0}$, $\mathbf{r_{g0}}$, $\boldsymbol{\varepsilon_0}$, $\mathbf{k_{tt0}}$, $\mathbf{X_{C0}}$, $\boldsymbol{\phi_0}$, $\mathbf{d_{V0}}$).

To compare the two types of modeling is enough to calculate the dependence $\mathbf{r_g/r = f(\varepsilon)}$ from:

$$\frac{G_{V,g}}{G_V} = \frac{\frac{2}{3} \cdot \varepsilon^2 \cdot A_f}{N_{por} \cdot \frac{\pi \cdot d^2}{4}} \cdot \frac{v_g}{v} =$$

$$= \frac{\left(\frac{2}{3}\right)^5 \cdot \left(\frac{\varepsilon^2}{1-\varepsilon}\right)^2}{\frac{\pi}{4} \cdot \left(\frac{4\sqrt{3}}{3} \cdot X_C + 1 - X_C\right) \cdot \left[\left(\frac{2\sqrt{3}}{3} - 1\right) \cdot X_C + (\sqrt{2}-1) \cdot (1-X_C)\right]^4} = \frac{r}{r_g} \qquad (37)$$

Some significant values for these ratios were calculated all over the domain of $\boldsymbol{\varepsilon_C < \varepsilon < \varepsilon_a}$. A graphical representation of $\mathbf{G_{V,g}/G_V(X_C)}$ and $\mathbf{r_g/r(X_C)}$ is shown in figure 2. A fourth degree polynomial in $\mathbf{X_C}$ could be obtained by regression:

$$\frac{r_g}{r} = X_C^4 - 3.2277 \cdot X_C^3 + 1.0968 \cdot X_C^2 + 1.15 \cdot X_C + 0.9345 \qquad (38)$$

the correlation coefficient being unitary.

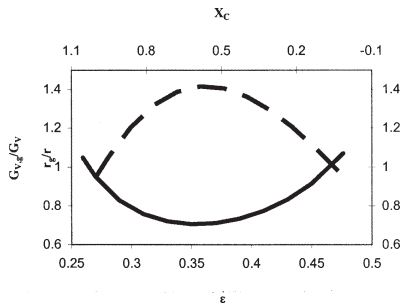

Fig. 2. Dependencies $G_{Vg}/G_V(X_C)$ —— and $r_g/r(X_C)$ - - from equation

Conclusions

The reference relations are given by the geometrical model, based on the ideal shape of deposed particles to form an incompressible precipitate and moreover on the degree of compaction for the layer. By weighting the two extreme degrees of compaction via X_C, the modified traditional model gives almost the same values like the geometrical model. Because a model based exclusively on the void fraction **ε** is more convenient (and nevertheless widely used already), the results obtained in this manner should be adjusted with the diagram from figure 2 or with relation (38). No other filtration techniques were analyzed (with variable pressure) because, for incompressible precipitates, the specific resistance **r** will be similar calculated. On the other hand, the problem of compressible precipitates remains open since the usual relation $r = r_1 \cdot (\Delta p)^s$ is purely empirical and presents a dimension deficit. An algorithm based on some relation of compaction growing during the filtration $X_C = f(t)$ is needed, either with $\Delta p =$ constant or with $G_V =$ constant. The increasing degree of compaction could be explained by particles' shape changing (they become flat), and also by changing of the characteristic dimension (the average diameter decreases). To develop a theory experimental data are necessary, concerning the variation of void fraction **ε** with **V** (and **t** consequently), at different values of Δp_{tot} in order to found a non-linear dependence $X_C(\varepsilon)$.

Nomenclature

A – surface (m^2);
a, b – constants in filtration equation (s / m^6, s / m^3);
d – diameter (m);
g – gravity (9.81 m / s^2);
G_m – mass flow (kg / s);
G_V –volumetric flow (m^3 / s);
h – height (m);
I – amperage (A);
k_{tt} – tortuosity factor;
ℓ, L – side, (characteristic) length (m);
m – mass (kg);
n, N – number;
Δp, p – (difference of) pressure (N / m^2);
q_{el} – current density (A / m^2);
q_m – mass flux ($kg / m^2 \cdot s$);
r – filtration specific resistance (m^{-2});
Re – ***Reynolds*** number;
R_f – filtration resistance (m / s);
S – generating line (m);
t – time, duration (s);
U – voltage (V);
v – flow rate (m / s);
V – volume (m^3);
w – mass fraction (kg / kg);
x – length (m);
X_C – degree of compaction;
Greeks:
δ – layer thickness (m);
ε– void fraction (m^3 / m^3);
μ – dynamic viscosity ($kg / m \cdot s$);
π – invariant of circle;
ρ – density (kg / m^3);
ρ_{el} – specific resistance (Ù · m);
σ – specific surface (m^2 / m^3);
ϕ – shape factor;
χ – volumetric ratio of precipitate;
ψ – sphericity.

Subscripts:
a – loose;
A – area;
C – compact;
calc – calculated;
cc – circle;
cil – cylinder;
con – cone;
dr – rectangle;
ech – equivalent;
f – filter, filtration;
g – voids, gaps;
hex – equiangular hexagon;
i – intermediate;
j – monodispersed fraction;
p – particle;
pat – square;
por – pore;
pr – precipitate;
ptg – equiangular pentagon;
rb – rhomb;
sc – circular sector;
solid – solid;
strat – layer;
susp – suspension;
S – sphere;
t – time;
tech – equilateral triangle;
tot – overall;
V – volume;
0 – initial, fictive, filter material (support)

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