Photonic Transmission of a Circular Aperiodic Superlattice Made of GaAs and Metamaterial Equivalent of NaCl

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The article analyzes the transmission properties of a circular aperiodic superlattices in the visible light range. The tests were carried out for the metamaterial equivalent of NaCl, which was defined as material A and GaAs - material B. Structure with the generation number of the L = {3, 4} circular superlattices was analyzed for P and S type of polarization. The structures were made of 250 nm and 300 nm layers thickness. The tests showed differences in transmission depending on the polarization. Also shown is the effect of increasing the thickness of the layers on the transmission.

Keywords: photonic, transfer marix, aperiodic, multilayers, metamaterials

Modern materials manufacturing technologies allow for structure design [1-4]. The application of successive layers of different materials allows the creation of a multi-layer structure with specific properties. One of the most interesting properties of multilayer structures is the occurrence of the phenomenon of photonic band gap. In materials characterized by the occurrence of this phenomenon, electromagnetic waves of specific frequencies do not propagate. This is used in scientific fields such as solid state physics, optoelectronics, photonics and optics.This allows the design and production of materials such as optical fiber photonic [5], photonic crystals [6-12], quasi crystals [13-20] and multilayer structures [21-26].

In 1968, Veselago predicted the theoretical possibility of materials with a negative refractive index [27]. In 2000, his theory was confirmed and the first metamaterial - left-handed material (LHM) was made [28]. The properties of these materials and multilayers systems were intensively tested [29-38]. Analysis of the numerical results of transmission values allows for designing structures with specific parameters. Among many methods that allow for the analysis of the properties of photonic multilayer systems, Transfer Matrix Method is often used [39]. In this algorithm, the characteristic matrix M is defined as follows

$$M=\begin{pmatrix} M_{1,1} & M_{1,2} \\ M_{2,1} & M_{2,2} \end{pmatrix}=D_{in}^{-1}\left[\prod_{i=1}^{N} D_i P_i D_i^{-1}\right]D_{out}. \tag{1}$$

D is the transmission matrix between layers, and P is the propagation matrix in a given i layer in the N layered structure.

From the characteristic matrix, the transmission T of the system can be determined

$$T=\frac{n_{out}\cos\Theta_{out}}{n_{in}\cos\Theta_{in}}\left|\frac{1}{M_{1,1}}\right|^2 \tag{2}$$

and reflectance R as

$$R=\left|\frac{M_{2,1}}{M_{1,1}}\right|^2. \tag{3}$$

n is refractive index described as

$$n=\sqrt{\varepsilon_r\mu_r}. \tag{4}$$

where ε_r is the relative permittivity of the electric medium and μ_r is the relative permittivity of the magnetic medium.The subscripts *in* and *out* are for the input and output layers of the structure, respectively.

The transmission matrix from layer a to b can be described by

$$D_{a,b}=\frac{1}{t_{a,b}}\begin{pmatrix} 1 & r_{a,b} \\ r_{a,b} & 1 \end{pmatrix}. \tag{5}$$

where t is Fresnel transmission coefficient and r is Fresnel reflection coefficient which for P and S polarization can make one of the forms

$$t_{a,b}^{P}=\frac{2}{1+\sigma_{a,b}^{P}}=\frac{\frac{2k_{a,x}}{\varepsilon_a}}{\frac{k_{a,x}}{\varepsilon_a}+\frac{k_{b,x}}{\varepsilon_b}}=\frac{\frac{2k_a\cos\Theta_a}{\varepsilon_a}}{\frac{k_a\cos\Theta_a}{\varepsilon_a}+\frac{k_b\cos\Theta_b}{\varepsilon_b}}=\frac{\frac{2n_a\cos\Theta_a}{\varepsilon_a}}{\frac{n_a\cos\Theta_a}{\varepsilon_a}+\frac{n_b\cos\Theta_b}{\varepsilon_b}}=\frac{2\sqrt{\frac{\mu_a}{\varepsilon_a}}\cos\Theta_a}{\sqrt{\frac{\mu_a}{\varepsilon_a}}\cos\Theta_a+\sqrt{\frac{\mu_b}{\varepsilon_b}}\cos\Theta_b}=\frac{2}{1+\sqrt{\frac{\mu_b\varepsilon_a}{\mu_a\varepsilon_b}}\frac{\cos\Theta_b}{\cos\Theta_a}} \tag{6}$$

$$t_{a,b}^{S}=\frac{2}{1+\sigma_{a,b}^{S}}=\frac{\frac{2k_{a,x}}{\mu_a}}{\frac{k_{a,x}}{\mu_a}+\frac{k_{b,x}}{\mu_b}}=\frac{\frac{2k_a\cos\Theta_a}{\mu_a}}{\frac{k_a\cos\Theta_a}{\mu_a}+\frac{k_b\cos\Theta_b}{\mu_b}}=\frac{\frac{2n_a\cos\Theta_a}{\mu_a}}{\frac{n_a\cos\Theta_a}{\mu_a}+\frac{n_b\cos\Theta_b}{\mu_b}}=\frac{2\sqrt{\frac{\varepsilon_a}{\mu_a}}\cos\Theta_a}{\sqrt{\frac{\varepsilon_a}{\mu_a}}\cos\Theta_a+\sqrt{\frac{\varepsilon_b}{\mu_b}}\cos\Theta_b}=\frac{2}{1+\sqrt{\frac{\mu_a\varepsilon_b}{\mu_b\varepsilon_a}}\frac{\cos\Theta_b}{\cos\Theta_a}} \tag{7}$$

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$$r_{a,b}^{P}=\frac{1-\sigma_{a,b}^{P}}{1+\sigma_{a,b}^{P}}=\frac{\frac{k_{a,z}}{\varepsilon_a}-\frac{k_{b,z}}{\varepsilon_b}}{\frac{k_{a,z}}{\varepsilon_a}+\frac{k_{b,z}}{\varepsilon_b}}=\frac{\frac{k_a\cos\Theta_a}{\varepsilon_a}-\frac{k_b\cos\Theta_b}{\varepsilon_b}}{\frac{k_a\cos\Theta_a}{\varepsilon_a}+\frac{k_b\cos\Theta_b}{\varepsilon_b}}=$$
$$=\frac{\frac{n_a\cos\Theta_a}{\varepsilon_a}-\frac{n_b\cos\Theta_b}{\varepsilon_b}}{\frac{n_a\cos\Theta_a}{\varepsilon_a}+\frac{n_b\cos\Theta_b}{\varepsilon_b}}=\frac{\sqrt{\frac{\mu_a}{\varepsilon_a}}\cos\Theta_a-\sqrt{\frac{\mu_b}{\varepsilon_b}}\cos\Theta_b}{\sqrt{\frac{\mu_a}{\varepsilon_a}}\cos\Theta_a+\sqrt{\frac{\mu_b}{\varepsilon_b}}\cos\Theta_b}= \quad (8)$$
$$=\frac{1-\sqrt{\frac{\mu_b\varepsilon_a}{\mu_a\varepsilon_b}}\frac{\cos\Theta_b}{\cos\Theta_a}}{1+\sqrt{\frac{\mu_b\varepsilon_a}{\mu_a\varepsilon_b}}\frac{\cos\Theta_b}{\cos\Theta_a}}$$

$$r_{a,b}^{S}=\frac{1-\sigma_{a,b}^{S}}{1+\sigma_{a,b}^{S}}=\frac{\frac{k_{a,z}}{\mu_a}-\frac{k_{b,z}}{\mu_b}}{\frac{k_{a,z}}{\mu_a}+\frac{k_{b,z}}{\mu_b}}=\frac{\frac{k_a\cos\Theta_a}{\mu_a}-\frac{k_b\cos\Theta_b}{\mu_b}}{\frac{k_a\cos\Theta_a}{\mu_a}+\frac{k_b\cos\Theta_b}{\mu_b}}=$$
$$=\frac{\frac{n_a\cos\Theta_a}{\mu_a}-\frac{n_b\cos\Theta_b}{\mu_b}}{\frac{n_a\cos\Theta_a}{\mu_a}+\frac{n_b\cos\Theta_b}{\mu_b}}=\frac{\sqrt{\frac{\varepsilon_a}{\mu_a}}\cos\Theta_a-\sqrt{\frac{\varepsilon_b}{\mu_b}}\cos\Theta_b}{\sqrt{\frac{\varepsilon_a}{\mu_a}}\cos\Theta_a+\sqrt{\frac{\varepsilon_b}{\mu_b}}\cos\Theta_b}= \quad (9)$$
$$=\frac{1-\sqrt{\frac{\mu_a\varepsilon_b}{\mu_b\varepsilon_a}}\frac{\cos\Theta_b}{\cos\Theta_a}}{1+\sqrt{\frac{\mu_a\varepsilon_b}{\mu_b\varepsilon_a}}\frac{\cos\Theta_b}{\cos\Theta_a}}$$

Electromagnetic wave propagation matrix P is definedas

$$P_i=\begin{pmatrix} e^{i\frac{2\pi f\cdot n_i\cos\Theta_i}{c}d_i} & 0 \\ 0 & e^{-i\frac{2\pi f\cdot n_i\cos\Theta_i}{c}d_i}\end{pmatrix}. \quad (10)$$

where θ_i is an angle to the normal structure in the layer i, f is the frequency of the incident wave and d_i is the i layer thikness.

Construction of a circular structure

In a circular superlattice, use the three-letter rule to generate a structure

$$\begin{aligned} A &\rightarrow CAC \\ B &\rightarrow ACCAC \cdot \\ C &\rightarrow ABCAC \end{aligned} \quad (11)$$

Then, for two materials with refractive indices n_A and n_B, a binary sequence should be generated using the rule

$$\begin{aligned} A &\rightarrow A \\ B &\rightarrow A \cdot \\ C &\rightarrow B \end{aligned} \quad (12)$$

The structure of the circular superlattices before and after the binary transformation was collected in table 1 and table 2, respectively.

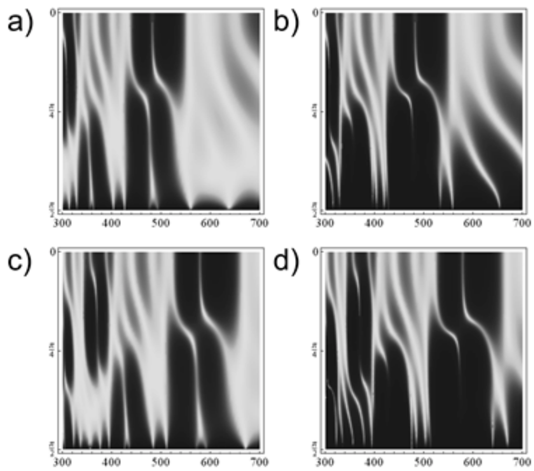

Fig. 1. Transmission of circular multilayer for L = 3 generation numer; the polarization type P (a, c) and S (b, d); thickness of the single layer is equal to 250 nm (a, b) and 300 nm (c, d)

Discussions

The transmission properties of a circular aperiodic superlattices in the wavelength range from 300 nm to 700 nm was analyzed. The material A was defined as the metamaterial equivalent of NaCl with the refractive index $n_A = -1,544$, and GaAs was material B with $n_B = 3.4$. Circular superlattices was analyzed for S and P polarization type.

The layers thickness in structures was 250 nm and 300 nm. Structure with the generation number of the L = {3, 4} was investigated. The results were collected in figures 1 and 2. White means full transmission while black means no transmission. On the horizontal axis of the drawings, the electromagnetic wavelength of the incident wave was determined. On the vertical axis, on the other hand, the

Table 1

THE STRUCTURE OF THE CIRCULAR SUPERLATTICES BEFORE THE BINARY TRANSFORMATION

Generation number L (number of layers)	The structure
1 (1)	A
2 (3)	CAC
3 (13)	ABCACCACABCAC
4 (55)	CACACCACABCACCACABCACABCACCACABCACCACACCACABCACCACABCAC

Table 2

THE STRUCTURE OF THE CIRCULAR SUPERLATTICES AFTER THE BINARY TRANSFORMATION

Generation number L (number of layers)	The structure
1 (1)	A
2 (3)	BAB
3 (13)	AABABBABAABAB
4 (55)	BABABBABAABABBABAABABAABABBABAABABBABABBABAABABBABAABAB

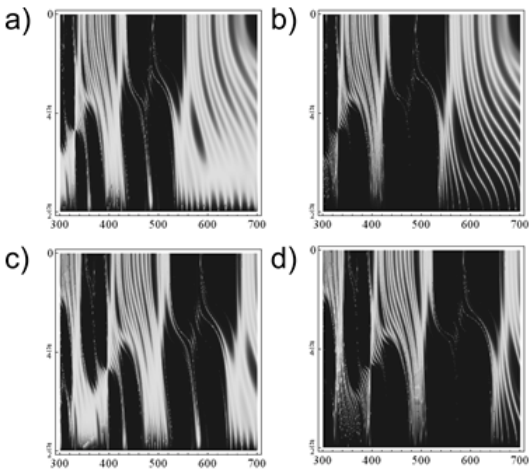

Fig. 2. Transmission of circular multilayer for L = 4 generation numer; the polarization type P (a, c) and S (b, d); thickness of the single layer is equal to 250 nm (a, b) and 300 nm (c, d)

angle of incidence of the wave in relation to the normal of the structure.

Conclusions

As can be seen in figures 1 and 2, the transmission of the electromagnetic wave passing through a multilayer structure consists of bands. The shape of the bands varies with the angle of incidence of the electromagnetic wave. When the thickness of the layers is increased, the structure of the transmission bands shifts towards the larger wavelengths. The shape of bandgaps for various structure generation numbers is similar. For L = 4, we can clearly see an increase in the number of transmission bands and a simultaneous reduction of their half width. For the studied structures, the effect of polarization on the electromagnetic wave transmission is not large but visible.

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